Astronomy Proficiency Exam 2019
Galaxies and the Universe (406)
Aug 23, 2019 — 9:00–11:00 AM

1. **Chemical Evolution [20 pts].** Consider a (10 pc)^3 cube within the Galactic disk which contains 35 $M_\odot$ of stars and 15 $M_\odot$ of gas. Assume we can treat the region as a closed box that was initially pure gas.

   (a) [**7**] If the current gas metallicity in the region is $1.2\,Z_\odot$, determine the nucleosynthetic yield in terms of $Z_\odot$.

   (b) [**5**] Assuming the yield you obtained in part (a) is always constant, determine the value of the gas metallicity at which the mass of metals in the ISM ($\mathcal{M}_h$) undergoes a transition from increasing ($\delta\mathcal{M}_h > 0$) to decreasing ($\delta\mathcal{M}_h < 0$).

   (c) [**3**] Does the transition described in part (b) occur in the past or the future, relative to the current time?

   (d) [**5**] If a different volume within the Galaxy contains equal masses of gas and stars at the present time, what value would you predict for its gas metallicity, assuming the same yield you obtained in part (a)?

2. **Spherical Model Galaxy [20 pts].** Giant elliptical galaxies tend to lie at the centers of rich clusters. Suppose we model a giant elliptical as a Plummer sphere,

$$\Phi(r) = -\frac{GM_{\rm tot}}{\sqrt{r^2 + a_0^2}}$$

   with a scale radius of a_0=1 kpc and a total mass $M_{\rm tot} = 10^{12}\ M_\odot$.

   (a) [**7**] Calculate the density profile of the galaxy model, $\rho(r)$.

   (b) [**7**] Calculate the total potential energy of the model,

$$U = \frac{1}{2}\iiint_V \rho(\mathbf{x})\Phi(\mathbf{x})\,dV\ .$$

   Hint: The following definite integral may be useful:

$$\int_0^\infty \frac{x^2\,dx}{(x^2+1)^3} = \frac{\pi}{16}\ .$$

   (c) [**6**] Use the virial theorem to estimate the central velocity dispersion of the galaxy in km s^{-1}.

3. **Spiral Galaxy.** A face-on spiral galaxy disk lies at a distance of 10 Mpc. It has an exponential scale length of 1 arcmin and a central surface brightness in V of 21 mag arcsec^{-2}.

   (a) [**8 pts**] Convert the exponential scale length to pc and the central surface brightness to $L_{V,\odot}$ pc^{-2}.

(b) [**6 pts**] By integrating from the center to infinite radius, calculate the total V-band luminosity of the galaxy in units of $L_\odot$.

(c) [**6 pts**] Calculate the absolute and apparent V magnitudes of the galaxy as a whole.

4. **High-redshift quasar.** We observe a quasar at redshift $z = 7.5$ in a deep sky survey. For simplicity we will assume a flat, matter-dominated model ($\Omega_{m,0} = \Omega_0 = 1$).

   (a) [**4 pts**] What was the value of the scale factor when the light we see today was emitted by the galaxy?

   (b) [**6 pts**] What was the age of the universe when the light we see today was emitted by the galaxy? You may express your answer as a multiple of $H_0^{-1} \approx 14$ Gyr.

   (c) [**6 pts**] Determine the distance modulus of the quasar, $m - M$.

   (d) [**4 pts**] If the mass of the black hole is 10^8 $\mathrm{M}_\odot$, what is the expected luminosity of the quasar in solar luminosities, if it is accreting at the Eddington rate?

5. **Horizons and Distances.** Suppose that we lived in a radiation-dominated universe where $\Omega_{r,0} = \Omega_0 = 1$. You can assume the standard value for H_0, so that $c/H_0 \approx 4.4$ Gpc.

   (a) [**7 pts**] By substitution of an appropriate expression for the scale factor $a(t)$, derive an algebraic expression for the proper distance as a function of z and H_0 for this model.

   (b) [**7 pts**] In this model, what would be the minimum possible angular diameter (in arcsec) of a galaxy with a true diameter of 10 kpc?

   (c) [**6 pts**] What is the horizon distance (in other words, the proper distance to an object at infinite redshift) in this model? How does it compare to the Hubble radius?