NAME:______________________________ **NETID:**____________________

Astronomy 404 Proficiency Exam
August 2020

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1. DO NOT OPEN THIS EXAM UNTIL INSTRUCTED TO DO SO.
2. Write you name and all answers on this exam. Turn in the exam and your study sheet, but note that the study sheet will not be graded.
3. Show all of your work in the test booklet, and indicate clearly your final answer! A correct final answer may not receive credit if no work is shown.
4. Budget your time! Don't get stalled on any one question.
5. The exam is closed book and closed notes, but you may use your 1-page study sheet.
6. For your reference there are constants listed below.
7. The total number of points on both parts of the exam is 200.

Possibly Useful Constants

Astronomical Unit: $1\,\mathrm{au} = 1.5 \times 10^{11}\,\mathrm{m} = c \times 500$ sec.
year: 1 yr $= 3.16 \times 10^7$ sec
parsec: $1\,\mathrm{pc} = 3.1 \times 10^{16}\,\mathrm{m} = 2.1 \times 10^5\,\mathrm{au}$
gravitational constant : $G = 6.7 \times 10^{-11}\ \mathrm{m^3\,kg^{-1}\,s^{-2}} = 6.7 \times 10^{-8}\ \mathrm{cm^3\,g^{-1}\,s^{-2}}$
speed of light: $c = 3.0 \times 10^8\ \mathrm{m\,s^{-1}} = 3.0 \times 10^5\ \mathrm{km\,s^{-1}} = 1.023\ \mathrm{pc\,Myr^{-1}}$
Boltzmann constant: $k = 1.38 \times 10^{-23}$ Joule/K $= 1.38 \times 10^{-16}$ erg/K $= 8.62 \times 10^{-5}$ eV/K
Stefan-Boltzmann constant: $\sigma_{\mathrm{SB}} = 5.7 \times 10^{-8}\ \mathrm{W\,m^{-2}\,K^{-4}}$
Wien's Law constant: $b = 2.9 \times 10^{-3}$ m K
Planck's constant: $h = 6.6 \times 10^{-34}$ J s $= 4.1 \times 10^{-15}$ eV s
electron Volt: 1 eV $= 1.6 \times 10^{-19}$ J
Boltzmann constant:
mass of the proton: $m_p = 1.7 \times 10^{-27}\,\mathrm{kg} = 0.938\,\mathrm{MeV}/c^2$
solar mass: $M_\odot = 2.0 \times 10^{30}$ kg
solar (total) luminosity: $L_\odot = 3.8 \times 10^{26}$ Watt $= 3.8 \times 10^{33}\ \mathrm{erg\,s^{-1}}$
solar radius: $R_\odot = 7 \times 10^8$ m $= 7 \times 10^{10}$ cm
present age of Sun $\sim 5 \times 10^9$ yr

1. *HR Diagrams*: **[30 points total]**

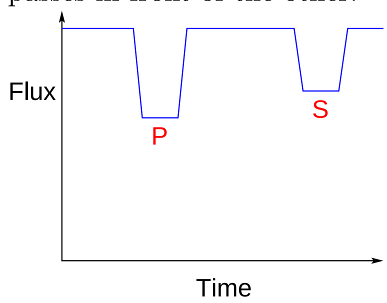
   (a) **[10 points]** Sketch an HR diagram for the life history of the Sun, from the zero age main sequence to the final end state.

   Clearly label the axes, and indicate which direction is increasing.

   (b) **[10 points]** On your diagram, indicate the region(s) where nuclear fusion *does occur* in the star's *core.* Label these "core fusion."

   (c) **[10 points]** On your diagram, indicate and name the region where there are *two* burning shells are active.

2. *The Light Curve of Eclipsing Binary Stars.* **[30 points total]** A binary star system is observed, with the orbit plane seen exactly edge-on. The resulting light curve of flux versus time is as shown below. for one period. We see that this is an eclipsing binary–each star periodically passes in front of the other.



(a) **[10 points]** Explain why the light curve shows that one star is **completely eclipsed** by the other.

(b) **[10 points]** Draw a sketch to show the orientation of the two stars relative to the observer for eclipse P. Be clear which is the larger star, and which is closer to the observer.

Now draw a similar sketch for eclipse S. Be clear which is the larger star, and which is closer to the observer.

(c) **[10 points]** Based on the light curve, which star has a **higher luminosity**–the one with the larger or smaller radius? Briefly explain why.

3. *The First Stars* **[30 points total]** The first stars formed after the Big Bang are expected to be made of only hydrogen, helium, and lithium when they are formed. That is, the first stars should be free of heavy elements.

   Yet *no* metal-free stars have ever been observed.

   (a) **[10 points]** Assume the lack of metal-free stars today is because no such stars have survived for the 13.6 Gyr age of the Universe.

   Therefore calculate the mass of a star with a main-sequence lifetime 13.6 Gyr. Express your answer in units of $M_\odot$.

   (b) **[10 points]** Is your answer to part (a) an upper limit or a lower limit to the masses of the first stars? Briefly explain.

   (c) **[10 points]** Based on your answers to (a) and (b), how does the mass distribution of the first stars compare with the mass distribution of stars observed today?

4. *Stellar Nucleosynthesis.* **[20 points total]** Stars are the dominant sources of most of the elements in the periodic table.

   (a) **[10 points]** Give the main reaction(s) by which stars produce carbon in their core.

   (b) **[10 points]** During which phase will a $1M_\odot$ star produce carbon?

5. *Black holes.* **[30 points total]** A brave astronaut, Captain Gaga, hovers above a black hole a radius $r_{\rm CG} = 2R_{\rm Sch}$, where $R_{\rm Sch}$ is the black hole's Schwarzschild radius.

   We at Mission Control observe the astronaut from a great distance much larger than the Schwarzschild radius, which we can take as $r_{\rm MC} \to \infty$.

   (a) **[10 points]** Due to a bureaucratic mix-up, Captain Gaga must take the ASTR 404 proficiency exam. She uses the full 3 hours, as measured by her clock.

   Calculate how long Captain Gaga appears to take to complete her exam, as seen from Mission Control.

   (b) **[10 points]** Captain Gaga fires a laser at us. We at Mission Control observe the laser photon at a wavelength of 707 nm. Calculate the wavelength of the *emitted* photons.

   (c) **[10 points]** Now consider *two* black holes with identical masses. In orbit around each, an astronaut hovers at radius $r = 2R_{\rm Sch}$, while carrying a clock designed to tick once in the time $\Delta t = 1$ sec. The two astronauts send signals *to each other*.

   How does each astronaut observe the tick rate of the other's clock, compared to her own? Briefly explain.

6. *The Next Supernova in Our Galaxy.* **[20 points total]** The massive star Betelgeuse explodes as a supernova.

   (a) **[10 points]** What will be our first indication of the explosion: photons or neutrinos? Briefly explain why.

   (b) **[10 points]** The supernova cools as it expands. Imagine the cooling links temperature and blast radius as $T \propto 1/R$. If the blast radiates thermally in this phase, should the luminosity increase, decrease, or stay the same with time?

   Justify your answer with a brief calculation.

7. *Stellar Structure* **[50 points total]** Consider a star in hydrostatic equilibrium. The star has radius R, and a pressure profile

$$P(r) = P_{\mathrm{c}} \left(1 - \frac{r^2}{R^2}\right) \tag{1}$$

with P_{c} a constant, the central pressure.

(a) **[10 points]** This pressure profile decreases with increasing r. Briefly explain why this must be the case for any star in hydrostatic equilibrium.

(b) **[10 points]** Use the fact that the star is in hydrostatic equilibrium, and the pressure profile in eq. (1), to show that the *product* of density with enclosed mass obeys $\rho(r)\, m(r) \propto r^3$.

(c) **[10 points]** Given your result for part (7b), show that the density ρ is a constant independent of r.

(d) **[10 points]** Assume our star obeying eq. (1) has a constant density ρ, and is made of a non-relativistic *ideal gas*. Now consider the temperature profile $T(r)$ with radius r.

- Find an expression for the central temperature T_{c} of the star in terms of the parameters give above and physical constants.

- Find an expression for $T(r)/T_{\mathrm{c}}$.

- Sketch a plot of $T(r)/T_{\mathrm{c}}$ from $r = 0$ to $r = R$.

- Interpret physically the temperature profile your have derived: where is the star hottest and coolest, and why?