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Spin of a batted baseball

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Abstract

Experiments are conducted to investigate the spin of a baseball undergoing an oblique collision with a bat. A baseball was fired horizontally at speeds up to 120 mph onto a 3"-diameter cylinder of wood that was rigidly attached to a wall. In one experiment, a two-wheel pitching machine was used in which the backspin or topspin of the incident ball could be adjusted. In another experiment, an air cannon was used to project the ball with no spin. In both experiments, markers on the ball were tracked with high-speed video to determine the velocity and spin vectors, before and after the scattering. Our primary results are as follows: (1) For a given angle of incidence, the scattered spin is nearly independent of the incident spin; (2) The spin of the scattered baseball is considerably larger than expected for a model whereby the ball rolls before leaving the surface. Implications for the spin of batted baseballs will be explored.

Keywords: Baseball; spin

1. Introduction

One of the major unsolved problems in baseball physics is our understanding of the spin of a batted ball. It is more than just a physics problem; it also has great practical value as knowing it would greatly enhance our ability to predict the flight of a baseball given the initial velocity vector. Low-speed ball-bat collisions have shown that the often-made sliding-to-rolling assumption is not valid [1]. Instead, the ball grips the surface of the bat, stretching the ball in the transverse direction, resulting in a significantly higher spin than can be obtained by rolling. The object of the present experiment is to extend the previous studies to much higher speed.

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2. Experiment and Data Reduction

Two different experiments were performed, both with the geometry shown in Fig. 1. A baseball is projected horizontally onto a three-inch diameter wood cylinder that is rigidly bolted to a rigid surface so that no recoil is possible. The incident velocity vector $\mathbf{v_1}$ is offset from the centerline of the cylinder by an amount E so that it makes an angle θ with the normal to the surface of the cylinder. The ball scatters through an angle α and has final velocity $\mathbf{v_2}$. The incoming and outgoing spins are ω_1 and ω_2, respectively, with the positive direction along the outward normal to Fig. 1, corresponding to incident topspin and outgoing backspin. In the first experiment, the ball was projected with a two-wheel pitching machine for which the incident speed and spin were independently adjusted to be in the range 85-110 mph and 1000-3000 rpm, respectively. In the second experiment, the ball was projected without spin at a fixed speed of 120 mph from a pressurized air cannon. In both experiments, the scattering was viewed by a video camera operating at 2000 frames/sec. Care was taken to assure the camera axis was normal to the scattering plane. A calibration grid in the field of view of the camera was used for calibration.
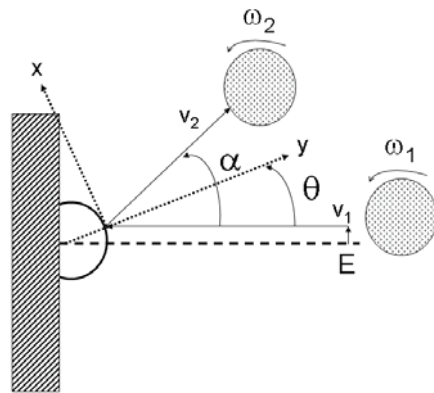

Fig. 1. Geometry for the scattering experiments, with the arrows indicating the positive direction for the various quantities

For the air cannon experiment, each baseball had four reflective markers that could be tracked with the high-speed video and the pixel coordinates determined for approximately 20 frames (0.01 sec). The camera calibration allowed the pixels to be converted to coordinates. One marker was placed near the rotation axis; the others were 0.9-1.3 inch from the axis. Using the formalism described in [2], a least-squares fitting procedure was used to fit simultaneously the locations of all markers in all frames to determine $\mathbf{v_1}$, $\mathbf{v_2}$, ω_1, ω_2, α, and the orientation of the spin axis. It was confirmed that the spin axis coincided with the normal to the scattering plane to better than 3 degrees. Neither θ nor E are directly measured. A similar but slightly different procedure was used for the other experiment. A total of 52 impacts were analyzed.

3. Ball-Bat Collision Model

The goal of the analysis is to interpret the experimental results in the context of the collision model described in detail by Cross [3]. First we denote x and y as directions transverse and normal to the surface of the bat at the contact point, respectively (see Fig. 1). The model is described by three parameters. One of these is the coefficient of restitution (COR) for the normal velocity component,

$e_y = -v_{y2}/v_{y1}$, which is approximately 0.5 for a baseball on a rigid wood surface. Another is the tangential COR e_x, defined as the negative ratio of final to initial tangential surface velocities of the ball:

$$e_x = -\frac{v_{x2} - r\omega_2}{v_{x1} - r\omega_1} \qquad (1)$$

where r is the ball radius. The third is the perpendicular distance from the center of the ball to the normal line of force, denoted by D:

$$\left(I\omega_2 + mv_{x2}r\right) - \left(I\omega_1 + mv_{x1}r\right) = -mD(1 + e_y)v_{y1} \;, \qquad (2)$$

where m and $I=0.4mr^2$ are the mass and moment of inertia of the ball. The left-hand-side of Eq. 2 is the difference between the final and initial angular momentum of the ball about the contact point, so that D=0 implies angular momentum conservation in the collision. Eqs. 1 and 2 can be combined to obtain expressions for the final spin and transverse velocity:

$$\omega_2 = \left(\frac{0.4 - e_x}{1.4}\right)\omega_1 + \left(\frac{1 + e_x}{1.4}\right)\frac{v_{x1}}{r} - \frac{D}{r}\left(\frac{1 + e_y}{1.4}\right)\frac{v_{y1}}{r} \qquad (3)$$

$$v_{x2} = \left(\frac{1 - 0.4e_x}{1.4}\right)v_{x1} + 0.4\left(\frac{1 + e_x}{1.4}\right)r\omega_1 - \frac{D}{r}\left(\frac{1 + e_y}{1.4}\right)v_{y1} \qquad (4)$$

The primary parameter determining the final spin of the ball is e_x, so it is useful at this stage to discuss its physical significance [3]. For ease of exposition, we consider the simplified situation with no initial spin and with D=0, in which case $\omega_2 = (5/7)(1 + e_x)v_{x1}/r$, which clearly shows the dependence of the final spin on e_x. When the ball makes contact with the bat, the latter exerts a normal force N on the ball in the y direction. If the ball has nonzero tangential velocity $v_{x1} - r\omega_1$, it will slide along the surface of the bat so that a frictional force $F=\mu N$, where μ is the coefficient of sliding friction, acts in the opposite direction to retard the tangential velocity. We consider three cases. First is the situation where F brings the sliding to a halt prior to the ball leaving the surface, in which case $e_x=0$ and the ball rolls along the surface as it leaves the bat. Second is the situation where F is insufficient to bring the sliding to a halt before the ball leaves the bat, a condition referred to as "gross slip". In this case the final and initial tangential velocities have the same sign so that $e_x<0$ and the spin is reduced relative to the rolling case. The gross slip and rolling cases are the only possibilities for a rigid baseball and were the only ones considered in [4,5]. However, for a baseball with tangential compliance, a third case is possible in which the ball grips the surface while still sliding, as elastic energy is stored in the tangential stretching of the ball. To analyze such a situation in detail requires a dynamic model [6,7]. Depending on the details the resulting tangential COR can be positive, so that the final spin is enhanced relative to the rolling case, a condition referred to as "overspin". The low-speed experiment of [1] found a modest overspin, $e_x=0.16$.

Eqs. 3-4 together with the definition of e_y give a complete description of the scattering process. For given initial conditions, there are three unknown parameters, e_x, e_y, and D, that can be determined from three observables v_{x2}, v_{y2}, and ω_2. A complication arises because θ is not measured experimentally so that the x and y components of the velocity vectors are not known. However from Eq. 4 and the definition of e_y, the following equation is derived

$$\tan(\alpha - \theta) = \left(\frac{1 - 0.4e_x}{1.4e_y}\right)\tan(\theta) + 0.4\left(\frac{1 + e_x}{1.4e_y}\right)\frac{r\omega_1}{v_1 \cos(\theta)} - \frac{D}{r}\left(\frac{1 + e_y}{1.4e_y}\right) \tag{5}$$

which can be solved numerically for θ given the measured quantities and the model parameters. The incident angle θ is largely determined by the scattering angle α and is only weakly dependent on the model parameters. The goal of the analysis is to find values of e_y, e_x, and D that best describe the collision data. To that end, a least-squares fitting procedure [1] was applied to the full set of data from 47 of the 52 impacts. The impacts not included were those with |α|>90°, since these show clear evidence for gross slip, as discussed in the next section. For the gross slip cases, e_x is no longer independent of the initial conditions, and the relationship between the final and initial tangential surface velocities is given by [3]

$$\frac{v_{x2} - r\omega_2}{v_{y1}} = \frac{v_{x1} - r\omega_1}{v_{y1}} - \left(3.5\mu - 2.5\frac{D}{r}\right)\left(1 + e_y\right) \tag{6}$$

4. Results and Discussion

The results of the experiment and analysis are given in Fig. 2, and the fitted parameters for the non-slip data are e_y=0.52, e_x=0.30±0.02, and D=0.0 mm. In Fig. 2(a) the final spin normalized to 100 mph incident velocity, $\omega_{2,norm} = \omega_2(100 \text{ mph}/v_1)$, is plotted as a function θ. The line is the expected result for e_x=0.3, D=0, and no initial spin. Some interesting observations follow directly from this plot. First, the spins are quite large, up to about 5000 rpm. Second, all the data for angles below about 30° fall on the same linear trajectory, regardless of whether the incident ball has backspin, topspin, or no spin, demonstrating that the final spin does not depend strongly on the initial spin. This results is easily understood from Eq. 3, since the coefficient multiplying ω_1 is small for e_x=0.3. Third, the spin saturates at about 5000 rpm for incident angles larger than about 40°, corresponding to scattering angles greater than 90°, demonstrating clear evidence for gross slip. Fig. 2(b) shows that the data are consistent with D=0 for |θ|<20°. Moreover, for |θ|<40° D/r is less than 0.05, corresponding to D<2 mm. It is concluded that angular momentum conservation is an excellent approximation over most of the angular range of the experiment. Fig. 2(c) is a plot of the final vs. the initial tangential surface velocity, normalized to the initial normal velocity. These quantities are related by the multiplicative factor $-e_x$. The dashed curve is the expected result for e_x=0.3 and is consistent with the data for absolute values of the abscissa less than about 0.7, corresponding θ=35° for impacts with ω_1=0. For larger impact angles, gross slip sets in and e_x rapidly passes through zero and changes sign. The dotted curve, the expected result for gross slip with μ=0.15 (Eq. 6), nicely accounts for that part of the data. This value of μ is significantly smaller than the lower limit of 0.50 found in the low-speed studies [1] and is not presently understood. Gross slip is further explored in Fig. 2(d), in which the ratio of the transverse to normal impulse imparted to the ball is plotted as a function of the normalized incident surface velocity. The curve is the expected ratio for e_x=0.3 and D=0, provided the ratio is less than μ, which is satisfied for values of the abscissa less than about 0.7. For larger values, gross slip occurs and the impulse ratio saturates at the value μ≅0.15. The self-consistency among these plots gives us confidence in our interpretation of the data.

We next investigate the implications of our results for batted baseballs, requiring modifications to the formalism for a free (rather than clamped) bat that is swung (rather than at rest), as developed in [1]. In the calculation, a baseball is projected horizontally at 85 mph onto a bat that is swung horizontally at 70 mph. The impact parameter E (see Fig. 1) is varied and the post-impact velocity and spin are calculated using our formalism and the fitted parameters. Fig. 3 shows ω_2 vs. the launch angle α for ω_1=±2000 rpm

and for e_x=0.3 (solid) and 0 (dashed), respectively.   For e_x=0.3, the two solid curves are essentially indistinguishable for impacts for which there is no gross slip, showing that the batted ball spin is not strongly dependent on the pitched ball spin for a given launch angle α.   This conclusion is very different from that found with e_x=0 [4,5], as seen from the dashed curves. Moreover, for pitched balls thrown with backspin (ω_1<0), the spin of a batted baseball with positive launch angle is considerably larger with e_x=0.3 than with e_x=0.
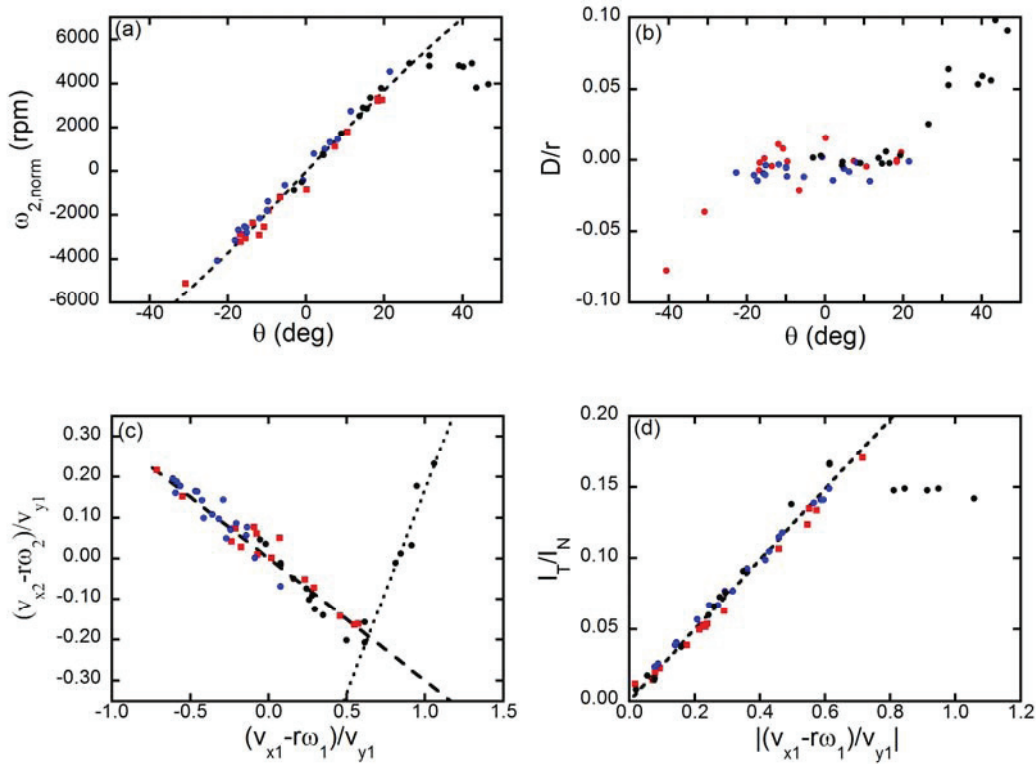

Fig. 2. Results of our analysis, with red, blue, and black points corresponding to incident backspin, topspin, and no spin, respectively.   (a) Scattered spin, normalized to 100 mph initial speed, vs. θ. The curve is the expected result for e_x=0.3 and D=0; (b) D/r vs. θ; (c) Final vs. initial tangential surface velocity, normalized to vy1. Dashed curve is expected result for e_x =0.3; dotted curve is result for gross slip with μ=0.15; (d) Ratio of transverse to normal impulses vs. the normalized initial transverse surface velocity saturates at the value $\mu \cong 0.15$. The dotted curve in Fig. 2(c) is the expected result for gross slip with μ=0.15

5. Summary and Conclusions

New experiments were conducted to investigate the spin of a baseball undergoing an oblique collision with a clamped cylinder. We summarize our findings as follows:

- For incident angles less than about 40°, the ball grips the surface and rebounds with considerable overspin, characterized by a tangential COR e_x=0.30±0.02.
- For incident angles less than about 30°, the angular momentum of the ball about the initial contact point is conserved in the collision

- For incident angles exceeding 40°, gross slip occurs and the corresponding coefficient of friction is 0.15.
- The spin of a batted baseball is less dependent on the spin of the pitched baseball less than previously thought based on the rolling scenario.
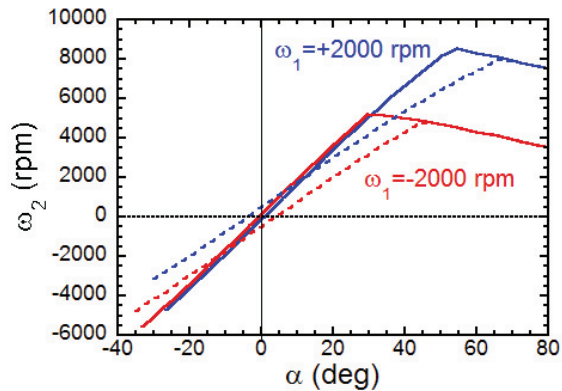

Fig. 3. Calculations of batted ball spin as a function of vertical launch angle under typical game conditions. The solid and dashed curves are for $e_x=0.3$ and 0, respectively; blue and red curves are for incident topspin and backspin, respectively

Acknowledgements

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