EFFECT OF BALL PROPERTIES ON THE BALL-BAT COEFFICIENT OF RESTITUTION

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A theoretical model is presented that relates the ball-bat coefficient of restitution e to the ball coefficient of restitution e_0 and dynamic stiffness k. The model is used to develop a technique to normalize e to values of e_0 and k for a "standard ball." The efficacy of this normalization technique is demonstrated by comparison with experimental data. It is shown to be vastly superior to a widely used technique that is based on the physically unjustified assumption that the ratio e/e_0, commonly referred to as the Bat Performance Factor or BPF, is independent of both e_0 and k.

1 Introduction

In recent years, an effort has been under way to measure and regulate the performance of nonwood baseball and softball bats. The measurement technique involves projecting a ball from a high-speed cannon onto a stationary bat and measuring the speed of the ball both before and after the collision. From these measurements, a value can be derived for the ball-bat coefficient of restitution (COR) e, which is a measure of energy dissipation in the ball-bat system. If e is to be a meaningful metric of bat performance, it is necessary to control the properties of the balls used to measure it. One such ball property is e_0, the COR of the ball when colliding with a rigid object, which determines the fraction of compressional energy stored in the ball that is returned as kinetic energy. A second ball property is k, the effective spring constant or "dynamic stiffness" of the ball. For a given bat, the ball stiffness controls how the initial energy is partitioned between compressional energy stored in the ball and that stored in the bat. The larger the ball stiffness, the less compressional energy is stored in the ball, leading to less overall energy dissipation and larger e. This phenomenon is popularly known as the "trampoline effect."

Based on these general ideas, a highly-simplified theoretical model is constructed that describes the dependence of e on e_0 and k. This model is used to develop a technique to normalize e to values of e_{0S} and k_S for a "standard ball." The normalization technique is tested by applying it to experimental data taken at the bat testing facility at the Sports Sciences Laboratory at Washington State University. While not perfect, the technique is shown to be vastly superior to another widely used technique.

2 Theoretical Consideratons

2.1. Toy Model for the Ball-Bat Collision

The starting point is a two-spring model for the ball-bat collision, Fig. 1, which was previously developed by Cross as a model for the trampoline effect in the interaction of tennis balls with the racket strings (Cross, 2000). In this model, the ball and bat are each represented as masses on linear lossy springs, with force constants k_0 and k_1, respectively. We hereafter refer to k_0 as the "dynamic stiffness" of the ball. The two springs mutually compress each other, converting the initial center-of-mass (CM) kinetic energy entirely into compressional potential energy.

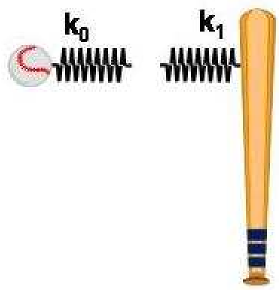

Figure 1. Simplified physical model for the ball-bat collision.

The fundamental equation for the energy dissipated in the collision is as follows:

$$1-e^2 = (1-e_0^2)f_0 + (1-e_1^2)f_1 \,, \tag{1}$$

where f_0 and f_1 are the fraction of the initial CM energy stored in the ball and bat, respectively; the quantities $(1-e_0^2)$ and $(1-e_1^2)$ are the fraction of stored energy that is dissipated in the ball or bat; and $(1-e^2)$ is the fraction of total CM energy that is dissipated in the collision. For linear springs, $f_0=k_1/(k_1+k_0)$ and $f_1=k_0/(k_1+k_0)$. Defining $r\equiv k_1/k_0$, which is the ratio of energy stored in the ball to that stored in the bat, Eq. 1 can be rearranged to obtain:

$$e^2 = \frac{re_0^2 + e_1^2}{1+r} \,. \tag{2}$$

Assuming no losses in the bat (i.e., e_1=1), a reasonable assumption for impacts near the sweet spot of the bat, then Eq. 2 can be rewritten to obtain Cross's result (Cross, 2000):

$$e^2 = \frac{re_0^2 + 1}{1 + r} \tag{3}$$

Eq. 3 is the basis for our normalization procedure.

A plot of e vs. r is shown in Fig. 2(a) for several different values of e_0. The limiting cases have simple physical interpretations. For r>>1, essentially all the CM energy is stored in the ball, none in the bat, and e approaches e_0, the value for the ball alone, essentially independent of r. This regime is typical of wood bats and low-performing hollow bats. In the opposite limit, r<<1, very little energy is stored in the ball, so that e approaches 1 (or e_1) independent of e_0. In the intermediate range, e is generally larger than e_0, as some of the energy that might have been stored and mostly dissipated in the ball is instead stored in the bat. For modern hollow metal or composite bats, r is generally in the range 2-15, a range in which e depends on the two ball properties, e_0 and k.

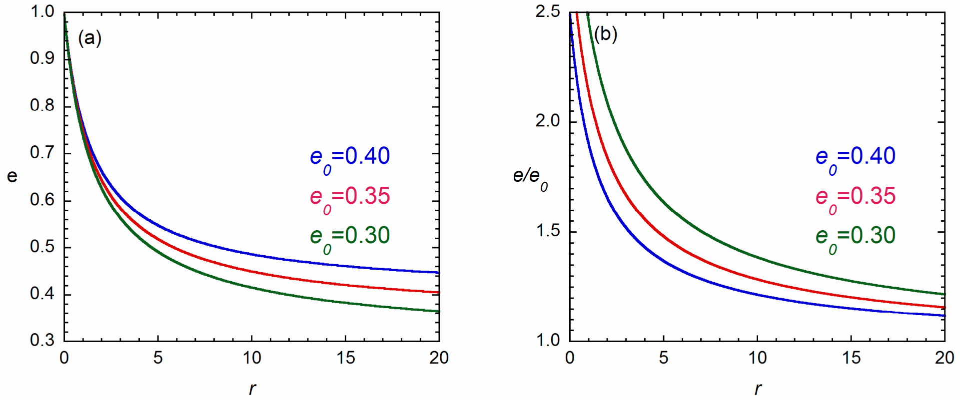

Figure 2. (a) Plot of e vs. r (Eq. 3) for three values of e_0. (b) Plot of the ratio e/e_0, commonly called the BPF, vs. r for three values of e_0, demonstrating that the BPF is not independent of either ball COR or dynamic stiffness. For the non-wood bat studied experimentally, $2.2 \le r \le 4.3$.

2.2 Normalizing to a Standard Ball

Suppose a ball of known COR e_0 and dynamic stiffness k is used to measure the ball-bat COR for a particular bat, obtaining e. Given that information, a technique is sought to predict the ball-bat COR e_S when the same bat is tested with a "standard" or normalizing ball S with COR e_{0S} and dynamic stiffness k_S. In the context of the two-spring model, an exact procedure can be obtained via Eq. 3. After some algebraic manipulation, our proposed normalization prescription is obtained:

two-spring model: $$e_S = \sqrt{\frac{k(1-e^2)e_{0S}^2 + k_S(e^2 - e_0^2)}{k(1-e^2) + k_S(e^2 - e_0^2)}}\ . \tag{4}$$

A different normalization procedure (Brandt, 1997) is widely used and is based on the assumption that the ratio e/e_0, commonly known as the Bat Performance Factor or BPF, is a property of the bat alone and independent of both e_0 and k. The BPF normalization is given by the formula

$$\text{BPF:} \qquad e_S = e_{0S}\,\frac{e}{e_0}\ . \tag{5}$$

However, the BPF assumption is not in general consistent with the two-spring model. Indeed, a careful inspection of Eq. 4 or Fig. 2(b) shows that that e/e_0 is independent of e_0 and k only in the limit $r>>1$, i.e., only for wood or low-performing hollow bats.

3 Experiment and Results

The bat and ball testing facility at the Sports Science Laboratory at Washington State University (Smith & Cruz, 2008; Smith, 2008) was used to study the dependence of e on the ball properties e_0 and k, with the specific goal of testing the normalization procedure of Eq. 4. The measurements consisted of firing a softball from an air cannon at 110±1 mph onto a stationary bat and measuring the incoming and rebound speed of the ball, from which the ball-bat COR is derived using standard formulas (Nathan, 2003). The measurements utilized 78 different standard softballs, whose COR and dynamic stiffness were determined in supplemental experiments (ASTM WK8910) and ranged from 0.31-0.39 and 5100-10,000 lb/inch, respectively. The balls were divided into groups of six, with balls in each group having nearly the same value of e_0 and k. The primary bat studied was a high-performing non-wood bat (Louisville Slugger Catalyst, 34 inches long, 26.5 oz). As we will discuss shortly, the r values for this bat and the balls used were in the range 2.2-4.3. From Fig. 2, we see that in this regime the ball-bat COR is a much stronger function of k than of e_0, whereas the BPF is a strong function of both k and e_0. This bat should therefore be particularly useful in distinguishing between the two normalization techniques. The impact location was fixed at 6.5 inches from the barrel tip. Additional data were taken on a wood bat (Brett Brothers Pro-Model 110, 33 inches long, 29 oz), for which the ball-bat COR is expected to be independent of k. After normalizing the wood bat COR to e_0 using Eq. 5, we confirm our expectations by finding the COR to be independent of k. The root-mean-square (rms) scatter of the normalized values about the mean is 0.005, which we take as an indication of the overall precision of our COR measurements.

The results of our study for the non-wood bat are presented in Fig. 3, where the plotted values are averages over the six balls in each group. Fig. 3(a) shows the dependence of e and e_S on k for balls with $0.36<e_0<0.37$, where e_S is calculated using Eq. 4, with normalizing values e_{0S}=0.36 and k_S=6700 lb/inch. The results show that e has a nearly linear dependence on k with a slope of 0.027 per 1000 lb/in. The slope is considerably reduced to 0.009 per 1000 lb/in by normalization, an improvement by a factor of three. Ideally the normalized slope would be zero, so there is some additional dependence of *e* on *k* that is not accounted for by the two-spring model. Fig. 3(b) shows the dependence on e_0 for balls with 6500<*k*<7000 lb/in. The two-spring normalization removes essentially all the dependence on e_0, reducing the slope of a linear fit by a factor of five. On the other hand, Eq. 5 overcorrects for e_0, resulting in a slope larger in magnitude and opposite in sign compared to the uncorrected data. The scatter plot in Fig. 3(c) of all the data shows that the large spread in unnormalized values of *e* is reduced considerably when the prescription of Eq. 4 is used to normalize. By comparison, the BPF normalization technique, Eq. 5, shows a spread comparable to the unnormalized values. Using Eq. 3, we estimate that the bat stiffness is approximately 22,000 lb/inch, so that *r* falls in the range 2.2-4.3. That the BPF technique works so poorly can be easily understood from Fig. 2(b), given the range of *r*. Indeed, the experimental BPF values are far from constant, ranging from 1.45-1.77 and 1.51-1.68 for the data in Fig. 3(a) and 3(b), respectively.

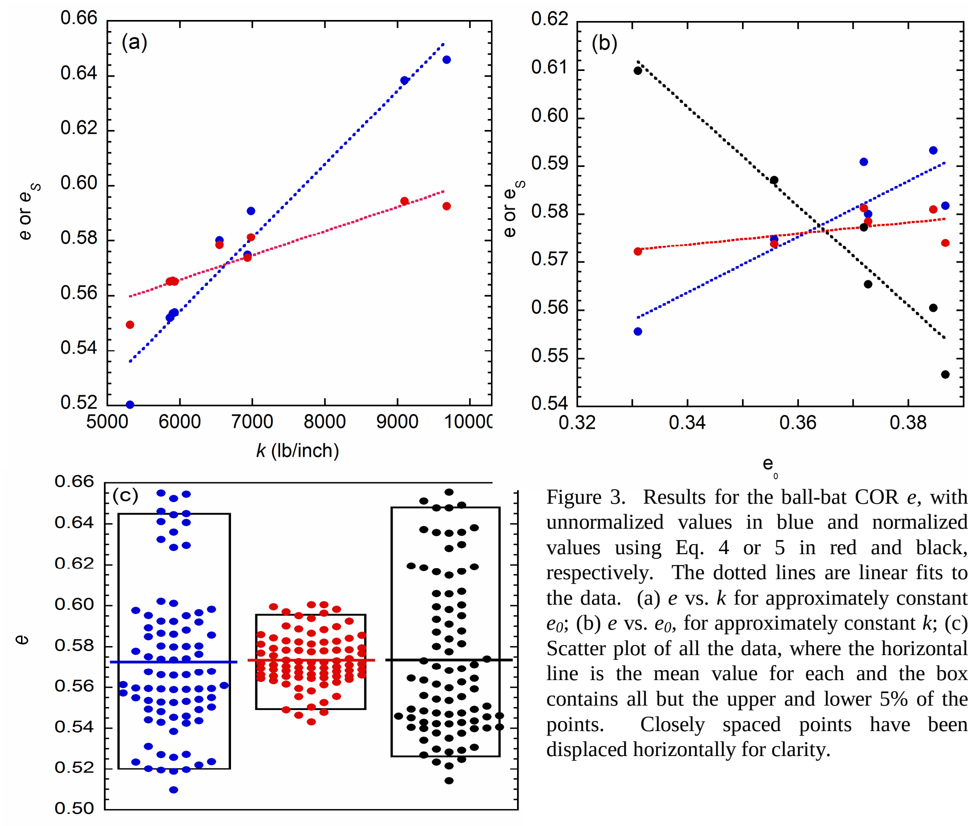

Figure 3. Results for the ball-bat COR *e*, with unnormalized values in blue and normalized values using Eq. 4 or 5 in red and black, respectively. The dotted lines are linear fits to the data. (a) *e* vs. *k* for approximately constant e_0; (b) *e* vs. e_0, for approximately constant *k*; (c) Scatter plot of all the data, where the horizontal line is the mean value for each and the box contains all but the upper and lower 5% of the points. Closely spaced points have been displaced horizontally for clarity.

4 A Useful Approximation

For low-performance bats, i.e., those with e not much larger than e_0, a useful approximation to Eq. 4 can be derived, taking advantage of the fact that $r>>1$, so that e should be nearly independent of *k:*

$$e_S \approx e_{0S} + e - e_0 \tag{6}$$

To demonstrate the effectiveness of the approximation, consider a bat with e=0.54 when measured with a ball of e_0=0.52. We normalize to a ball of the same k and e_{0S}=0.50, obtaining 0.5213 and 0.5200 using Eqs. 4 and 6, respectively, a difference of only 0.25%.

5 Summary

We have presented a model of the ball-bat collision that explicitly demonstrates the dependence of the ball-bat COR e on the COR e_0 and dynamic stiffness k of the ball. We have used this model to develop a technique for normalizing e to properties of a standard ball. We have tested the model with a high-performance softball bat for which there is a strong nearly linear dependence of e on k and have shown that the normalization technique, while not perfect, reduces that dependence by about a factor of three. We have further shown that the dependence of e on e_0 is removed by the normalization. We have shown experimentally that the ratio e/e_0, known as the BPF, depends on both e_0 and k, as predicted by the two-spring model. Therefore it is not surprising that the BPF normalization method fails for the non-wood bat tested. Finally we have derived an approximate normalization expression which is valid for low-performing bats.

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