Models of baseball bats

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By observing the vibrations of a hand-held baseball bat, it is possible to show that the bat behaves as if it were a free body at the impact of the bat and the ball. The hand-held bat shows none of the behavior of a bat with one end firmly clamped in a vise.

It has been conjectured, but never verified, in several articles in this Journal[1,2] that a hand-held baseball bat behaves as if it were a free body when it strikes the ball. Recently, Weyrich *et al.*[3] attempted to determine the effect of bat grip firmness on the post-impact velocity of a baseball. This was done using both a freely suspended bat and a bat with its handle firmly clamped in a massive vise. What was not done was to determine which of these two limiting cases more closely resembles the actual case of a hand-held bat, or whether a loosely held bat corresponds to a freely suspended bat and a tightly gripped bat corresponds to a bat in a vise. Such an investigation has been carried out for tennis rackets,[4] and rackets have been found to act as if they are free bodies when the ball impacts on them. However, a tennis racket is very flexible and quite light compared to a baseball bat, and in addition, it is usually held by just one hand. Therefore, it is not clear whether both implements will behave in the same way during the very short time that the ball is in contact with the bat or racket.[5]

This investigation shows that the freely suspended bat (essentially a free body) corresponds to both the loosely held and very tightly gripped bat, and a bat firmly clamped in a vise does not behave the way a hand-held bat does. It also shows how well the hands tend to damp out the vibrations of the bat that occur when the bat is struck at a location that is not near the node.

One of the basic differences in behavior between a free bat and a bat with one end firmly clamped is the allowable normal modes of oscillation of the bat. Figure 1 shows the lowest modes of oscillation for these two cases. For the clamped bat, the fundamental mode of oscillation (often called the "diving board" mode) has a node only at the clamp. The next higher mode of oscillation of the clamped bat has a node well beyond the center of mass of the bat in addition to the one at the clamp. For the free bat, the mode of oscillation corresponding to the fundamental mode of the clamped bat is not allowed. The lowest frequency of oscillation has two nodes, a period not greatly different from the period of the first harmonic of the clamped bat, and an outer node that is located near the position of the node of the first harmonic of the clamped bat. If a clamped bat is struck at an antinode of the first harmonic (such as the tip), both the low-frequency (diving board) mode and the higher-frequency first-harmonic mode should be present. If a free bat is struck at the same location (near the tip), because the low-frequency (single-node) oscillation is not excited, the lowest observed frequency should be the one corresponding to the two-node mode. Therefore, if a hand-held bat is struck near the tip and the low-frequency (single-node) oscillation is not observed, that would be good evidence that the hand-held bat is acting as if it were a free body. If the low-frequency mode of oscillation is observed in a hand-held bat, that would be good evidence that the bat was acting as if one end were clamped.

This experiment consisted of measuring the natural frequencies of oscillation of baseball bats that were free, clamped, and hand held with various degrees of firmness. It was then possible to determine which situation (free or clamped) more accurately described the hand-held bat, and how grip firmness affected this result. To accomplish this, a small Kynar thin-film piezoelectric vibration sensor[6] was taped to the handle of the bat, approximately 0.35 m from the butt end. The output from the Kynar was fed directly into an oscillosocpe and the resultant traces photographed. Two bats were used, an aluminum softball bat and a wooden softball bat. In each case the bat was struck by a ball at the desired location. Since the Kynar sensor produces a measurable voltage for a very small amplitude of vibration, it was possible to carry out the experiment by hitting the bat with a ball that was hand held rather than having to swing the bat at the ball. This allowed a certain degree of reproducibility in the results as well as introducing a modicum of safety into an experiment that you might want students to do.

The results obtained using the aluminum bat and the wooden bat were essentially the same, and so no distinction will be made between the two bats in describing the experiment and the conclusions drawn, even though there were substantial differences in the frequencies of vibrations of the two bats. Note that there is a major controversy in baseball concerning whether aluminum bats (which are not allowed in the Major Leagues) are better than wooden bats, but that problem will not be addressed in this article.

Figure 2 shows the output of the Kynar vibration sensor when a bat, with its handle firmly clamped in a vise, is struck with a ball near the node on the first harmonic and then struck near the tip. In both cases a low-frequency oscillation (27 Hz) is shown and, in addition, there is a higher-frequency (317 Hz) signal present in the off-node hit

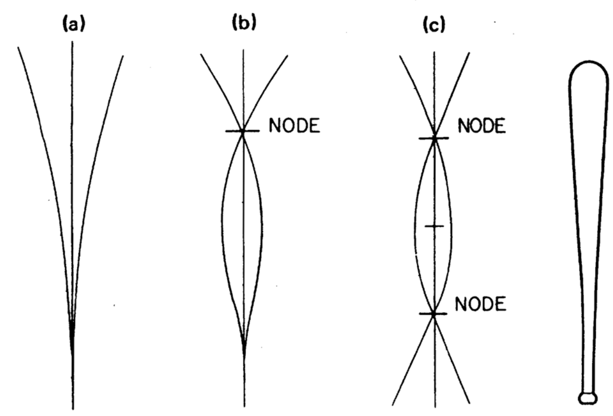

Fig. 1. Oscillations of a baseball bat (a) and (b) with one end clamped and (c) with both ends free.

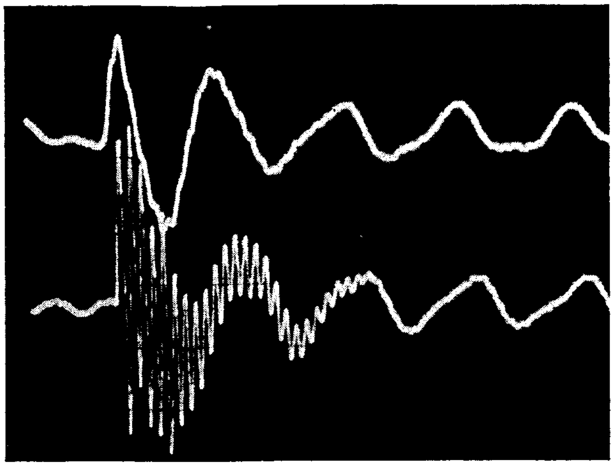

Fig. 2. Oscilloscope traces showing the output of a Kynar vibration sensor fastened to a bat with its handle clamped in a vise. The bat was struck near the node (upper trace) and near the tip (lower trace). The sweep speed was 20 ms/div (200 ms full scale), and the vertical gain of the scope was 0.2 V/div.

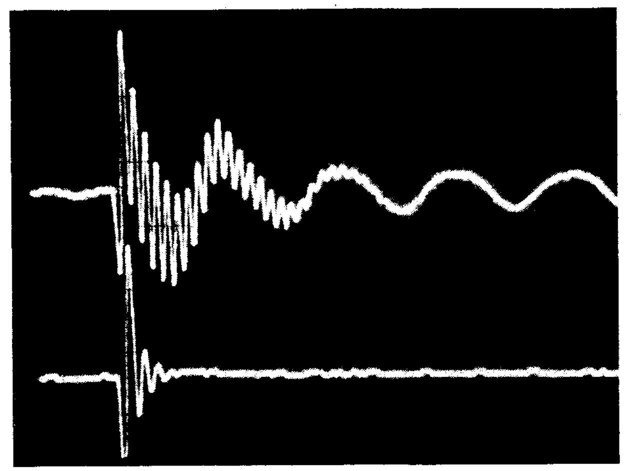

Fig. 4. Oscilloscope traces showing the output of Kynar vibration sensor fastened to a bat with its handle clamped in a vise (upper trace) and hand held (lower trace). In both cases the bat was hit near the tip, and the sweep speed was 20 ms/div with a vertical gain of 0.2 V/div.

that is much larger in amplitude than the higher-frequency oscillation that is present when the bat is struck near the node. The lower-frequency oscillation corresponds to the mode of vibration shown in Fig. 1(a), while the higher-frequency oscillation corresponds to the vibration shown in Fig. 1(b).

If the bat is freely suspended and hit with a ball near the tip, the oscillations produce the signal shown in the upper trace of Fig. 3. The frequency of this oscillation is about 20% lower than the high-frequency oscillation shown in Fig. 2 (clamped handle), and there is no visible sign of the low-frequency oscillation that is very obvious in Fig. 2. These free bat oscillations correspond to the vibrations shown in Fig. 1(c). The lower trace in Fig. 3 shows the same bat held in a vise and struck at the same location. It is also clear from these scope traces that the amplitude of the higher-frequency vibrations damp out quicker than the low-frequency (fundamental) oscillations when the bat has its handle clamped in a vise.

The oscillations of a hand-gripped bat are shown in the lower trace of Fig. 4. There is no sign of the low-frequency oscillation that is evident when the bat handle is in a vise, as is shown in the upper trace of Fig. 4. This means that a bat that is firmly held by hands acts as if it were a free bat, as far as the allowable modes of vibration are concerned. It is clear, by the rate at which the vibrations damp out, that the hands do affect the subsequent vibration of the bat. This can be seen quite easily in Fig. 5 where the oscillations of a bat that is loosely held and tightly gripped are shown. A number of additional pictures were taken at various sweep speeds of the oscilloscope and hitting the bats at various locations while gripping the handles very tightly. None of these pictures showed the low-frequency oscillation that was present when the bat handle was clamped in a vise.

The frequencies of oscillation, as determined from the scope traces, as well as other measured parameters of the

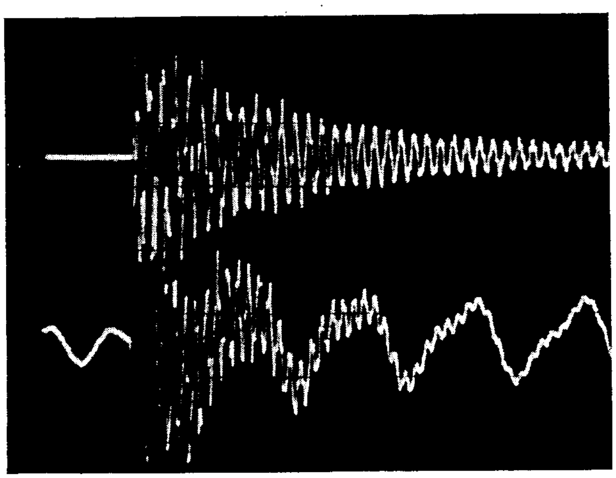

Fig. 3. Oscilloscope traces showing the output of Kynar vibration sensor fastened to a bat with both ends free (upper trace) and with its handle clamped in a vise (lower trace). The bat was hit near the tip in both cases, and the sweep speed was 20 ms/div with a vertical gain setting on the oscilloscope of 0.2 V/div.

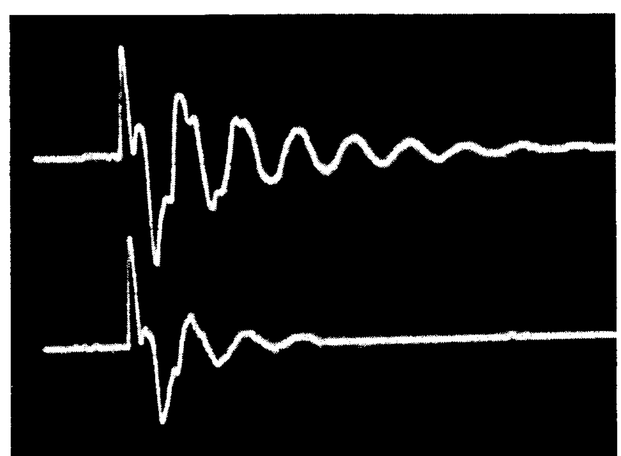

Fig. 5. Oscilloscope traces showing the output of a Kynar vibration sensor fastened to a bat that was hand held loosely (upper trace) and tightly gripped (lower trace). For these data, the sweep speed was 5 ms/div (50 ms full scale), and the vertical gain was 0.2 V/div.

Table I. Bat parameters.

	Aluminum bat	Wooden bat
Mass (kg)	0.825	0.846
Length (m)	0.81	0.84
Moment of inertia about cm (kg m^2)	0.046	0.047
Distance butt to cm (m)	0.46	0.51
Fundamental frequency of oscillation clamped handle (Hz)	27.2	18.2
First harmonic, clamped handle (Hz)	317	209
Lowest frequency free bat (Hz)	242	163
Deformation under 98-N load (m)	1.5×10^{-4}	4.2×10^{-4}

bat that may be of interest, are given in Table I. Since the measured length and inertial properties of the two bats are rather similar and their frequencies of vibration differ by 50%, the stiffness of the two bats must be quite different. Because one expects the natural frequency to be proportional to the square root of the stiffness, the aluminum bat should be more than at least twice as stiff as the wooden bat. It was noted that it required higher gain on the oscilloscope when the oscillations of the aluminum bat were being observed than when the oscillations of the wooden bat were being observed (0.2 vs 1 V/div), and this is a good indication that the aluminum bat is stiffer. To obtain a quantitative comparison, each bat was subjected to a downward force near its center with the ends of the bat blocked up off a table. Under comparable loads (98 N), a point on the wooden bat deformed approximately two and a half times as much as the corresponding point on the aluminum bat.

These tests were conducted with softball bats struck by softballs. When the same bats were struck with a hardball, the results led to the same conclusions—the hand-held bat behaves as if it were a free body. This is to be expected, since the contact time of a hardball on a bat has been determined to be about 1.5 ms (Ref. 5) compared to the 3.5-ms contact time of a softball on a bat.

These data lead to some interesting conclusions. Grip firmness at impact time should not influence the post-impact velocity of the ball. Grip firmness probably does have a significant influence on the bat velocity and bat position control up to the impact. However, if the batter were to release the bat completely at the time of impact, the subsequent trajectory of the ball should be the same as if the bat were firmly gripped throughout the swing. The subsequent trajectory of the bat would be quite different in these two cases, much to the dismay of the pitcher. In addition, the concept of adding some fraction of the hand and arm mass to the bat mass to get an effective mass or striking mass[7,8] seems to be contradicted by the results presented in this article.

[1]P. Kirkpatrick, "Batting the ball," Am. J. Phys. **31**, 606–613 (1963).

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[3]A. Weyrich, P. Messier, B. Ruhman, and M. Berry, "Effects of bat composition, grip firmness, and impact location on postimpact ball velocity," Med. Sci. Sports Exercise **21**, 199–205 (1989).

[4]H. Brody, "Models of tennis racket impacts," Int. J. Sports Biomech. **3**, 293–296 (1987).

[5]S. Plagenhoef, *Patterns of Human Motion* (Prentice-Hall, Englewood Cliffs, NJ, 1971), p. 71. Based on 4000-frame/s photography, the contact time of a softball on a bat was found to be 3.5 ms and the contact time of a tennis ball on the strings was 4–5 ms. A similar result for tennis balls was found using a laser and an oscillosocpe by H. Brody, Am. J. Phys. **47**, 482–487 (1979).

[6]Kynar piezoelectric film is available form Pennwalt Corp., P. O. Box C, King of Prussia, PA 19406.

David Griffing, *The Dynamics of Sports* (Dalog, Oxford, OH, 1987), 3rd ed., pp. 101–103.

[8]S. Plagenhoef, *Patterns of Human Motion* (Prentice-Hall, Englewood Cliffs, NJ, 1971), pp. 61–63.

The Hall effect in copper: An undergraduate experiment

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This experiment on the Hall effect uses only apparatus ordinarily found in an elementary laboratory, and the Hall effect device itself can be constructed easily from ordinary materials without special equipment. It illustrates the vector nature of the Lorentz force equation, shows that the charge carriers in copper are negatively charged, and permits determination of the charge-carrier density and other related parameters in ordinary commercial copper.

I. INTRODUCTION

The equation for the magnetic force on a moving charge,

$$\mathbf{F}=q\mathbf{V}\times\mathbf{B}, \tag{1}$$

is found in one form or another in all introductory physics texts along with discussions of elementary applications, such as calculating the side thrust on a wire, or as the basis for the deflection of a charged-particle beam. Frequently, the Hall effect[1] is introduced as an example of this relation.[2] In some texts, the Hall effect is used as the basis for calculations of various parameters such as the charge-car-