

# Effect of Spin and Speed on the Lateral Deflection (Curve) of a Baseball; and the Magnus Effect for Smooth Spheres

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(Received March 26, 1959)

The effect of spin and speed on the lateral deflection (curve) of a baseball has been measured by dropping the ball while spinning about a vertical axis through the horizontal wind stream of a 6-ft tunnel. For speeds up to 150 ft/sec and spins up to 1800 rpm, the lateral deflection was found to be proportional to the spin and to the square of the wind speed. When applied to a pitched ball in play, the *maximum* expected curvature ranges from 10 to 17 in., depending on the spin. The deflections of rough baseballs accord in direction with that predicted by the Magnus effect. But with *smooth* balls at low speeds the deflection is in the *opposite* direction. This is studied with an apparatus specially designed to measure the pressure at any point in the equatorial plane of the rotating ball.

## INTRODUCTION

EVERYONE who has played golf or baseball or tennis knows that when a ball is thrown or struck so as to make it spin, it usually "curves" or moves laterally out of its initial vertical plane.

How is this lateral deflection related to the spin and speed of the ball? An experimental answer to this question was sought for baseballs.

The first explanation of the lateral deflection of a spinning ball is credited by Lord Rayleigh<sup>1</sup> to Magnus,<sup>2</sup> from whom the phenomenon derives its name, the "Magnus effect."

The commonly accepted explanation is that a spinning object creates a sort of whirlpool of rotating air about itself. On the side where the motion of the whirlpool is in the same direction as that of the wind stream to which the object is exposed, the velocity will be enhanced. On the opposite side, where the motions are opposed, the velocity will be decreased. According to Bernoulli's principle, the pressure is lower on the side where the velocity is greater, and consequently there is an unbalanced force at right angles to the wind. This is the Magnus force.

In the case of a cylinder or a sphere, the so-called whirlpool, or more accurately the circulation, does not consist of air set into rotation by friction with the spinning object. Actually an

object such as a cylinder or a sphere can impart a spinning motion to only a very small amount of air, namely to that in a thin layer next to the surface. It turns out, however, that the motion imparted to this layer affects the manner in which the flow separates from the surface in the rear, and this in turn affects the general flow field about the body and consequently the pressure in accordance with the Bernoulli relationship. The Magnus effect arises when the flow follows farther around the curved surface on the side traveling with the wind than on the side traveling against the wind. This phenomenon is influenced by the conditions in the thin layer next to the body, known as the boundary layer, and there may arise certain anomalies in the force if the spin of the body introduces anomalies in the layer, such as making the flow turbulent on one side and not on the other. As we shall see, a reverse Magnus effect may occur for smooth spheres. Rough balls, such as baseballs and tennis balls, do not show this anomalous effect.

The ingenious experiments which led Magnus to the discovery of the effect were made chiefly with a small cylinder rotating about a vertical axis in a horizontal wind and so mounted that it was free to move laterally across the wind, but not downstream. The pull of a cord wrapped around the axis served to give the cylinder its initial spin. Magnus makes no comment about the smoothness of the surface of the cylinder. The boundary-layer concept, which was introduced by Prandtl in 1904, was of course not available to Magnus.

<sup>1</sup> Lord Rayleigh, "On the irregular flight of a tennis ball," Scientific Papers 1, 344 (1869-81).

<sup>2</sup> G. Magnus, "On the deviation of projectiles; and on a remarkable phenomenon of rotating bodies." Memoirs of the Royal Academy, Berlin (1852). English translation in Scientific Memoirs, London (1853), p. 210. Edited by John Tyndall and William Francis.

The beautiful photograph made by Professor F. N. M. Brown of the University of Notre Dame in his low-turbulence wind tunnel illustrates what has just been said (see Fig. 1). Here the wind with its smoke filaments is coming from the right at 60 feet per second. The ball is stationary in the tunnel but spinning counterclockwise at 1000 revolutions per minute about a horizontal axis at right angles to the wind. The crowding together of the smoke filaments over the *top* of the ball shows an increased velocity in this region and a corresponding decrease in pressure, which according to the Bernoulli principle, would tend to deflect the ball *upward* across the wind stream.

It will also be noted that the *wake* of the ball has been deflected *downward*. According to the principle of the conservation of momentum, this must likewise be accompanied by a corresponding *upward* thrust on the ball.

Put in other words, if the wind speed is from east to west and the ball is spinning counterclockwise about a *vertical* axis, then the Magnus force on the ball is directed towards the north.

For a further discussion of the flow past rotating cylinders, including many photographs, see Prandtl<sup>3</sup> and Goldstein.<sup>4</sup>

## PART I. EXPERIMENTS WITH BASEBALLS

### Air-Gun Experiments

My first measurements were made with an air-gun which had earlier been constructed at the National Bureau of Standards to measure the coefficient of restitution of baseballs.<sup>5</sup> The ball was mounted on a spinning tee located in front of the muzzle. The wooden projectile from the air-gun drove the spinning ball a distance of 60 ft (the distance from the pitcher's rubber to the home plate) where it made an imprint on a vertical target.

The spin of the ball before impact was measured with a Strobotac. The speed could (in theory) be computed by measuring the drop of the ball, i.e., the vertical distance of the projected

horizontal axis of the gun above the point of impact.

The direction of the spin about the vertical axis could be reversed at will and measurements were made with the ball first spinning clockwise (looking down) and then counterclockwise. One-half of the horizontal distance between the two target imprints gave the lateral displacement sought.

This setup was simple, and at first sight appeared usable. The observed deflections were in the expected direction, and shifted to the other side of the target when the spin was reversed. But the results were erratic. A stroboscopic camera was then installed 30 ft above the floor looking down on the last half of the flight path, and the position of the ball was photographed at exact 0.05-sec intervals against the scale on the floor.

These photographic measurements gave the trajectory of the ball, its speed and the drag; but they also indicated that the spin of the ball was greatly reduced when it was distorted by the impact of the projectile; and that the reaction between the spinning ball and the projectile gave rise to a small component of velocity normal to the flight path, which contributed to the observed lateral deflections. The trajectory was in fact that of a batted ball instead of a pitched ball. This line of attack was consequently abandoned in favor of wind tunnel measurements.

### Wind Tunnel Measurements

In these measurements, the spinning ball was dropped from the upper side of the NBS 6-ft



FIG. 1. Showing airflow past spinning ball in wind tunnel. Wind coming from right, 60 ft/sec. Spin 1000 rpm, counterclockwise, about a horizontal axis at right angles to wind. Magnus force, upward. Courtesy of Professor F. N. M. Brown, University of Notre Dame.

<sup>3</sup> Ludwig Prandtl, *Essentials of Fluid Dynamics* (Hafner Publishing Company, Inc., New York, 1952).

<sup>4</sup> S. Goldstein, *Modern Developments in Fluid Dynamics* (Clarendon Press, Oxford, England, 1938), Vols. 1 and 2.

<sup>5</sup> Lyman J. Briggs, *J. Research Natl. Bur. Standards* 34, 1 (1945).

octagonal wind tunnel across a horizontal wind of known velocity. By coating the bottom of the ball lightly with a lubricant containing lamp-black, its point of impact was recorded on a sheet of cardboard fastened to the tunnel floor. The lateral deflection, which is of immediate interest, was taken as one-half of the measured spread of the two points of impact, with the ball spinning first clockwise and then reversed.

The spinning mechanism was mounted outside on the top of the tunnel with its hollow shaft projecting vertically downward one-half inch through the tunnel wall. A concentric suction cup to support the ball was mounted on this shaft. The spinning ball was released by a quick-acting valve which cut off the suction and opened the line to the atmosphere.

The spinner was belt-driven by a small dc motor with its armature current supplied from a potentiometer circuit to secure the desired speed range. The angular speed (rpm) was measured with a calibrated Strobotac which illuminated a rotating target on the spinner mechanism. The ball was shielded by a thin-walled cylinder (4 in. o.d., 4 in. long) mounted on the inner wall of the tunnel, concentric with the spinner shaft. While this introduced some additional turbulence over that created by the bare ball on its spinner, it gave more consistent

TABLE I. Lateral deflection of a spinning baseball in a 6-ft drop across the tunnel windstream at various spins and speeds.

Spin rpm	Speed ft/sec	Deflection, inches	Ratio of deflections	Ratio of (speeds) <sup>2</sup>
1200	125	17.8	1.52	1.56
	100	11.7		
1200	150	26.0	1.46	1.44
	125	17.8		
1200	150	26.0	2.22	2.25
	100	11.7		
1200	100	11.7	1.92	1.77
	75	6.1		
1200	125	17.8	2.91	2.79
	75	6.1		
1200	150	26.0	4.25	4.0
	75	6.1		
1800	125	25.8	1.47	1.56
	100	17.5		
1800	125	25.8	2.98	2.79
	75	9.4		
1800	100	17.5	1.81	1.77
	75	9.4		

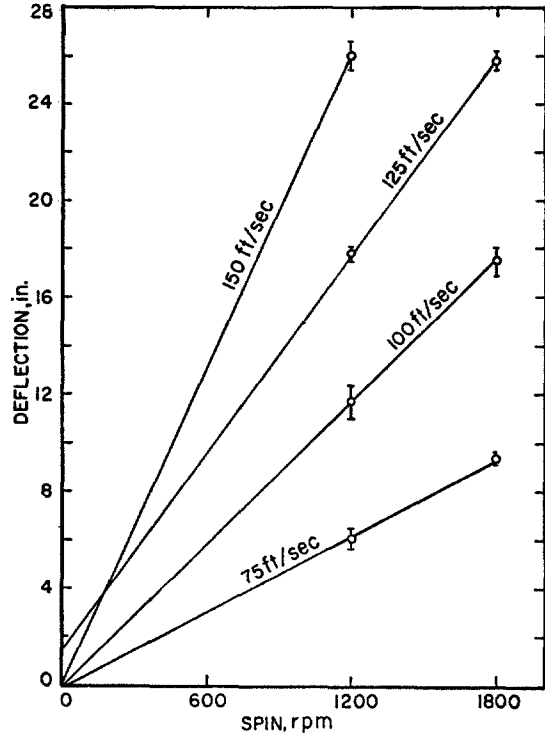


FIG. 2. Lateral deflection of a baseball, spinning about a vertical axis, when dropped across a horizontal windstream. These values are all for the same time interval, 0.6 sec, the time required for the ball to cross the stream.

results, and prevented irregularities caused by the ball being torn off its support by the wind stream during release. Official American League balls were used throughout the measurements.

Owing to the method of construction, the center of gravity of a baseball often does not coincide exactly with its geometrical center. As a result of this asymmetry, the ball rotating on its spinner is subject to a lateral centrifugal force. This causes the ball to depart from a truly vertical fall when there is no wind. The departure may be upstream, laterally, or downstream, depending on the angular position of the heavy side of the ball when released, and results in a scatter of impacts for the same spin and windspeed.

To minimize this effect, the ball was turned in different positions while being placed in the suction cup of the spinner, until a position in which the center of gravity appeared to fall on the spin axis was found by trial.

At least three measurements were made for each spin and wind speed, as well as for the spin reversed. The mean values are given in Table I

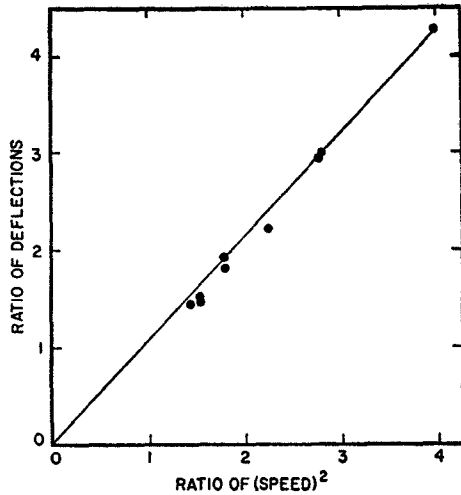


FIG. 3. Graph showing that the observed lateral deflections are proportional to the square of the wind speed.

and shown graphically in Fig. 2 together with the standard deviations. The lateral deflection in all instances accorded in direction with that expected from the Magnus effect.

It will be noted from Fig. 2 that straight lines drawn through the observed deflections at spins of 1200 and 1800 rpm for different wind speeds pass nearly through the origin. In other words, within experimental limits, the lateral deflection is directly proportional to the spin.

The effect of wind speed on the lateral deflection is shown in Table I. The fourth column of the table gives the ratio of the observed deflections at known wind speeds. For comparison, the last column gives the ratio of the corresponding wind speeds squared. These results are plotted in Fig. 3. Subject to experimental errors, the square relationship is seen to hold.

We conclude then that for speeds up to 150 ft/sec and spins up to 1800 rpm, the lateral deflection of a baseball spinning about a vertical axis is directly proportional to spin and to the square of the wind speed.

**Maximum Curve Expected for Pitched Baseballs in Play**

All the lateral deflections shown in Fig. 2 refer to what took place in 0.6 sec, the time required for the ball to fall across the wind stream of the 6-ft tunnel. We have now to convert these measurements into what deflection

would be expected if the ball were traveling the 60 ft from the pitcher's rubber to the plate at various speeds and spins.

The results are summarized in Table II. A ball thrown at a speed of 100 ft/sec would travel the 60 ft from rubber to plate in 0.6 sec, which is the time required for the ball to fall across the wind stream in the tunnel measurements. Consequently in this case the observed lateral deflection in the tunnel would be equal to that of the ball in play. In other cases a correction is necessary.

The lateral deflecting force of the tunnel stream is practically constant for a given spin and speed, so that the lateral acceleration of the ball is constant and the distance traveled is proportional to the square of the elapsed time. At 125 ft/sec, the thrown ball travels 60 ft in 0.48 sec, while the dropped ball takes 0.6 sec to cross the tunnel. Hence the deflection in 0.48 sec would be

$$17.8 \text{ in.} \times (0.48/0.60)^2 = 11.4 \text{ in.}$$

It will be noted from Table II that the amount the ball curves in 60 ft is proportional to the spin, but is practically independent of the speed, namely, about 11 in. at 1200 rpm and 17 in. at 1800 rpm. This result may seem surprising until it is recalled that at the higher speeds, the ball is in the 60-ft zone for a shorter time and that the lateral displacement is proportional to the time squared.

These measurements were all made with the ball spinning about a vertical axis, which gave the maximum lateral deflection. Usually in play the spin axis is inclined, which reduces the effect. If the spin axis were horizontal and normal to the flight path, no lateral deflection would take

TABLE II. Curving of baseballs in play.

Speed ft/sec	Spin rev/sec	Turns in 60 ft	Curve (lat. def.) in., in 60 ft
75	20	16	10.8
75	30	24	16.7
100	20	12	11.7
100	30	18	17.5
125	20	9.6	11.4
125	30	14.4	16.5
150	20	8	11.6

place. With clockwise spin (seen from right) the pitch would be a drop.

### Notes Pertaining to Baseballs in Play

These measurements were designed to cover simply the range of conditions encountered in play. The following records are of interest in this connection. Bob Feller, former Cleveland pitcher, in 1947 threw a baseball across the plate at a speed of 98.6 mph (144 ft/sec), as measured with electronic instruments. J. G. Taylor Spink, Editor of the *Sporting News*, states that this is the accepted world record for the fastest pitch.<sup>6</sup> The next fastest pitch of record is 94.7 mph (138 ft/sec) by Atley Donald, former New York Yankee pitcher, in 1939.

Dr. H. L. Dryden kindly arranged for the measurement of the "terminal velocity" of a baseball, that is its maximum speed after falling from a great height to the ground.<sup>7</sup> This measurement was carried out in a vertical wind-tunnel at the National Advisory Committee for Aeronautics, the wind speed being adjusted until the ball just floated in the windstream. The terminal velocity was about 140 ft/sec. The celebrated catch by Charles Street, of the Washington Ball Club, of a ball dropped from a window of the Washington Monument gave a computed velocity *in vacuo* of 179 ft/sec. The NACA measurements show that, owing to the resistance of the air, the actual speed could not have exceeded 140 ft/sec. However, home runs *batted* into the stands must have an *initial* velocity considerably higher than this.

With the cooperation of the pitchers of the Washington Ball Club, the spin of a pitched ball was measured. This was done by fastening one end of a long tape to the ball and then laying the rest loosely (but untwisted) on the ground between the rubber and the plate, the free end being pegged down. After the ball was caught, the number of turns was counted. The highest spins measured were 15.5–16 turns in 60 ft and the lowest 7–8. Assuming the speed of the pitch to be 100 ft/sec, the maximum spin measured would be about 1600 rpm. These spins are covered by the wind tunnel observations.

<sup>6</sup> J. G. Taylor Spink (personal communication).

<sup>7</sup> H. L. Dryden (personal communication, with permission).

### PART 2. SMOOTH BALLS AND THE MAGNUS EFFECT

The experiments with *rough* baseballs (Part 1) all showed lateral deflections in conformance with the Magnus effect. But with *smooth* balls, especially at low wind speeds, an anomaly is encountered. The deflections are usually in the *opposite* direction.

Maccoll, using a 6-in. diam wooden sphere, rotating on a wind tunnel balance, was apparently the first to demonstrate the existence of a small negative lift on a sphere at low wind speeds.<sup>8</sup> His  $C_L$  curve, negative at first, crosses the *no lift* axis when the equatorial speed/wind speed =  $U/V = 12.3/24.6 = 0.5$ .

Davies, in his experiments with smooth and dimpled golf balls, found that for the smooth ball the lift was negative at all rotational speeds below 5000 rpm (equatorial speed, 2100 ft/min).<sup>9</sup> Above this, the lift was positive, but was less than for the standard ball.

In Davies' measurements, the ball was dropped across the horizontal wind stream of an open tunnel operating at 105 ft/sec. The axis of spin was horizontal and normal to the wind stream. A quick-acting device released the spinning ball. The vertical drop was 0.67 to 1.3 ft. Spins up to 8000 rpm could be obtained.

It will be noted that in Davies' experiments the point of impact on the tunnel floor represents the combined effect of spin and drag, both being directed downstream. To get the lift, the point of impact had to be reduced by the drag with no spin; whereas in my baseball measurements the lateral deflection, being at right angles to the drag, could be measured directly.

Brown has recently measured the lift coefficient of a sphere of 3.36-in. diam mounted on a balance in a low-turbulence tunnel, and rotating at fixed speeds ranging from 700 to 4500 rpm.<sup>10</sup> His results all show a negative lift at low wind speeds, the graphs crossing the no-lift axis at values of  $V/U$  ranging from 0.1 to 0.5, where  $U$  is the wind speed and  $V$  is the peripheral speed.

<sup>8</sup> Maccoll, *J. Roy. Aeronaut. Soc.* **32**, 777 (1928).

<sup>9</sup> J. M. Davies, *J. Appl. Phys.* **20**, 821 (1949). This paper contains references to articles not here recorded.

<sup>10</sup> F. N. M. Brown, University of Notre Dame, Notre Dame, Indiana (personal communication, 1958; unpublished data, with permission).

TABLE III. Lateral deflection (inches) of smooth Bakelite ball, 6-ft drop.

Wind speed, ft/sec	Spin, rpm	Deflection, in.	
75	1200	1.1	Pro-Magnus
100	1200	0.5	Pro-Magnus
125	1200	0.6	Anti-Magnus
150	1200	2.2	Anti-Magnus
75	1800	1.4	Pro-Magnus
100	1800	0.5	Pro-Magnus
125	1800	0.8	Anti-Magnus
150	1800	7.3	Anti-Magnus

### Smooth Rubber Ball

The writer has measured the lateral deflection of a *smooth* rubber ball, using the same setup employed with baseballs. The ball, which had been cast in an accurately spherical mold, was 2.88 in. in diam, practically that of a baseball, but was heavier (wt 188 g; baseball about 145 g). Its center of gravity agreed closely with its geometrical center and it had a good bounce.

This smooth ball was deflected laterally *opposite* in direction to that of the baseballs. The deflection was small (partly owing to the increased weight) but increased steadily from 3.6 in. at 75 ft/sec (1800 rpm) to 8.8 in. at 150 ft/sec (all negative Magnus). At 1800 rpm the rotational velocity of a point on the equator was 1350 ft/min. Davies states that his smooth golf ball began to develop a positive lift at 5000 rpm (equatorial speed, 2100 ft/min).

### Bakelite Sphere

Similar tests were made with a smooth Bakelite sphere (good ground finish) of 3-in. diam, sphericity 0.005 in., wt 312 g. The Reynolds number at 150 ft/sec was  $2.4 \times 10^5$ . At this speed the ball was presumably passing out of the critical Reynolds-number range for the drag coefficient for spheres.<sup>11</sup> See Goldstein.<sup>4</sup>

The lateral deflections, which are very small, are given in Table III. The unexpected result was that at the lower wind speeds the ball had a Magnus deflection, crossing to the anti-Magnus regime between 100 and 125 ft/sec. Here we seem to have a new effect for *smooth* balls, which has been masked in the measurements reported above.

<sup>11</sup> The Reynolds number is the product of the speed of the wind and the diameter of the sphere divided by the kinematic viscosity of the air.

It seems now to be well established by the results of four different laboratories that a spinning *smooth* ball at low wind-tunnel speeds usually does not conform with Magnus effect but exerts a "negative" lift. It appears likely that this occurs in a certain Reynolds-number range where it is possible for the boundary layer on the side moving with the wind to remain laminar while that on the opposite side becomes turbulent. Since a turbulent layer will in general follow farther around the surface before separating than a laminar layer, the crosswind force is reversed, if the rate of spin is not too high. We would expect to pass from this condition into the Magnus regime when the spin is sufficiently increased or when the flow on both sides becomes turbulent, as it will when the Reynolds number is sufficiently increased.

Evidently we may also pass into the Magnus regime at low Reynolds numbers by arranging conditions so that the flow is laminar on both the approaching and receding sides, the requirements now being that disturbances, such as vibration of the sphere and turbulence in the wind stream, are sufficiently reduced. This is believed to be the explanation of the pro-Magnus results of Table III.

In general, then, we find that the sign and magnitude of the effect of spin are dependent on dynamic conditions as well as on the "smooth-

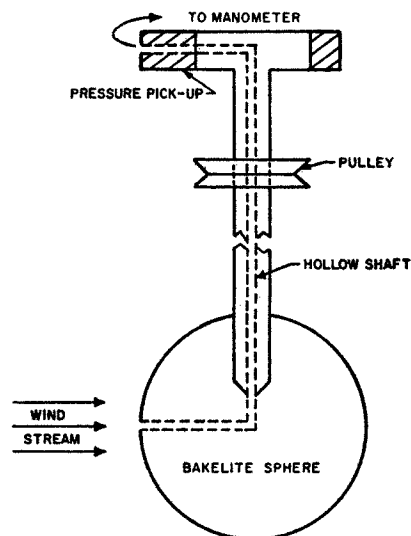


FIG. 4. Schematic drawing of apparatus used to measure the pressure at various points on a ball spinning in a wind stream.

ness" of the ball. The foregoing explanations have been offered as the most reasonable ones on the basis of the information available. The author knows of no detailed investigations of the flow in these cases.

### PART 3. PRESSURE DISTRIBUTION OVER A SPINNING SPHERE IN ITS EQUATORIAL PLANE

To learn more about the forces acting on the surface of a smooth rotating sphere, the following apparatus, shown schematically in Fig. 4, was constructed.

The smooth Bakelite sphere (3-in. diam) used earlier in the free-fall measurements was mounted on a hollow vertical shaft ( $\frac{3}{8}$ -in. diam), the center of the unshielded sphere projecting downward ( $2\frac{1}{2}$  in.) into the horizontal wind stream of the tunnel. A  $\frac{1}{8}$ -in. hole drilled along an equatorial radius connected the surface of the sphere with the hollow shaft.

The shaft extended upward through the top wall of the 6-ft octagonal tunnel, where it was expanded into a cylindrical head ( $1\frac{1}{8}$ -in. diam), which was drilled radially to lie directly above the hole in the sphere. The head was surrounded by a closely fitted pickup sleeve, which through a drilled radial hole ( $\frac{1}{16}$ -in. diam) connected the ball with the manometer. The pickup sleeve could be rotated and held independently in any angular position about its vertical axis, so that the pressure on the ball at any point in its horizontal equatorial plane could be measured.

The opposite leg of the manometer was connected to a static pressure orifice in the wall of the tunnel. The wind speeds (75 and 125 ft/sec) were based on impact pressure measurements in the empty tunnel. The hollow shaft was belt-driven by a dc motor on a potentiometer circuit. The spin, measured by a stroboscope, was roughly 1800 rpm.

As far as the writer knows, this is the first time the pressure at the equatorial surface of a spinning sphere has been measured directly. It provides a point-to-point determination of the pressure around the entire periphery of the sphere and eliminates the disturbance set up by the introduction of an external probe. By drilling the entrance hole in the sphere at higher latitudes, the pressure distribution over the entire

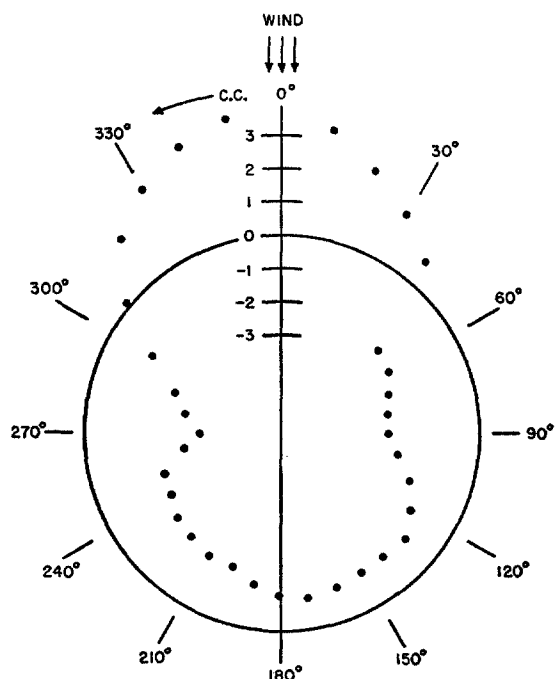


FIG. 5. Polar diagram of pressure distribution around smooth Bakelite ball (looking down along vertical spin axis). Ball, 3-in. diam; counter-clockwise spin, about 1800 rpm; wind speed, 125 ft/sec; pressure, inches of water.

surface of the sphere could be found. Only the equatorial pressures have been measured in the present study.

Two examples are given of the pressure distribution around the spinning sphere. In Fig. 5, the observed pressures at 125 ft/sec and 1800 rpm are shown in a polar diagram. It will be seen that the pressure is above atmospheric for  $40^{\circ}$ – $50^{\circ}$  on either side of the impinging windstream and that for this region the difference in pressure of symmetrical points would tend to force the ball to the right in Fig. 5, an anti-Magnus effect. The cosine projection of these above-atmospheric pressures on a horizontal diameter normal to the windstream is, however, small. For the remainder of the diagram, the pressures in the left half are consistently lower than corresponding points in the right half, tending to force the ball to the left, thus giving a positive Magnus effect.

If we sum up the pressure differences of corresponding points in Fig. 5 (after a cosine projection on a transverse equatorial diameter as illustrated in Fig. 6), we come out with a value of  $-5.78$  pressure units, the minus sign indicating that the resultant transverse force is in the

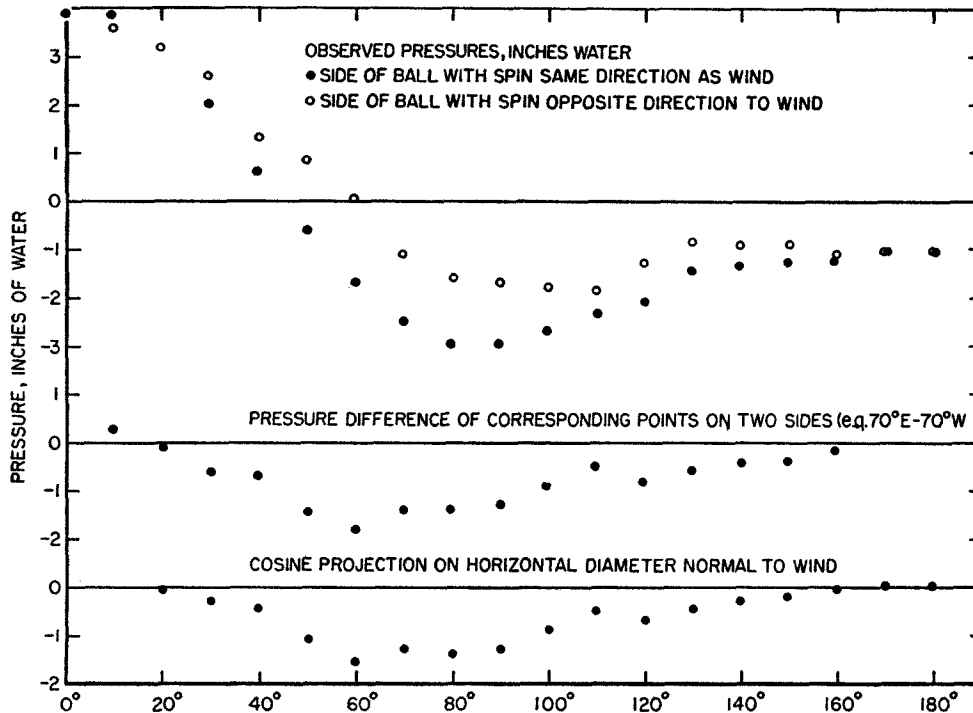


FIG. 6. Rough Bakelite ball, 3-in. diam; wind speed, 125 ft/sec; spin, about 1800 rpm. Meridional rubber bands stretched over ball to simulate roughness.

Magnus direction at 125 ft/sec and 1800 rpm. The corresponding measurement at 75 ft/sec is  $+1.77$  pressure units, an anti-Magnus effect. These results are opposite in sign from those obtained when the Bakelite ball was dropped across the windstream. Here, however, we are dealing only with the pressures in a narrow equatorial belt on the ball ( $\frac{1}{8}$ -in. wide) and not with its entire surface.

#### Rough Bakelite Ball

Finally, the surface of the same ball used in the preceding measurements was roughened by attaching rubber bands along meridional lines. Figure 6 shows: (a) the observed pressures at  $10^\circ$  intervals measured from the direction of the wind; (b) the pressure difference of correspond-

ing points; and finally (c) the projection of these differences on a horizontal diameter normal to the wind. The resultant pressures are consistently in accord with the Magnus effect, but it will be noted that this effect is substantially reduced by what takes place on the opposite side of the ball. Their sum,  $-10.8$ , taken in conjunction with appropriate units of area, gives a measure of the resultant force on the ball at the equator. The corresponding figure for the smooth Bakelite ball at the same speed is  $-5.78$ .

#### ACKNOWLEDGMENT

I would like to express my indebtedness to G. B. Schubauer, B. L. Wilson, R. H. Heald, R. J. Hall, and G. H. Adams for valued assistance in various ways.