THE PHYSICS OF BASEBALL

For almost a century and a half, baseball has played a significant role in defining the United States; in defining the physics of baseball we confront the ill-defined physics of the world in which we live.

Robert K. Adair

The illumination of the ordinary—of why the sky is blue or why the stars shine—is not the least important role of physics and physicists. Then can't we add to the list of deeper queries some of the questions that seemed so important to me in my youth: How can Babe Ruth hit so many home runs? What makes Carl Hubble's curveball and screwball swerve in their trips to the plate? And if baseball plays no known role in the fundamental structure of the universe (see *The Iowa Baseball Confederacy* by W. P. Kinsella[1] for a contrary position), it is not of trivial importance in the perception and appreciation of that universe by some of its inhabitants. Although not quite so important now, in the period between the Civil War and World War II baseball was a significant part of what defined the United States. Forty years ago, Jacques Barzun, a preeminent student of American culture and a native of France, said, "Whoever wants to know. . . America had better learn baseball."[2] But, even as the game itself is subtle and complex, I have found subtleties and complexities in my attempts[3] to know the physical bases of this American game.

Baseball, like golf and tennis, which also center on the flight of a ball struck by an implement, has important elements that can be addressed intelligently by a physicist. The aerodynamic forces on the ball as it passes through the air play an important role in the ball's flight and the character of the game. The laws of mechanics constrain the largely physiological character of the transfer of energy to the bat by the player, and they define the collision of the ball and bat. But the physics of baseball is not the clean, well-defined physics of fundamental matters but the ill-defined physics of the complex world in which we live, where elements are not ideally simple and the physicist must make best judgments on matters that are not simply calculable. The baseball is not uniformly smooth or rough but is characterized by the familiar yin–yang pattern of raised stitches. Moreover, the ball is not made of a uniform elastic substance but is constructed, following an ancient, arcane formula, from various kinds of wool yarn and cotton thread. And the bat is not a rigid cylinder with simple mechanical properties but a more complicated wooden figure with significant flexibility. Hence conclusions about the physics of baseball must depend on approximations and estimates.

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But estimates are a part of the physicist's repertoire. Enrico Fermi supposedly said that a competent physicist should be able to estimate *anything* to a useful degree. That supposed facility is put to the test when a physicist addresses sports: Little is precisely calculable, and much must be derived through intelligently constructed approximations. Moreover, there are too few reliable experimental data. Indeed, for baseball, some of the best data are derived from the game itself; what we see in the game constrains the physics of the game. Even as interested physicists should know the game, they should know the way the players play the game. After more than 100 years of trial and error, we must assume that what the experienced professional ballplayer *does* in playing the game is very nearly optimum. (See figure 1.) What the player *says* about the game should be taken seriously, but must often be reinterpreted.

And so, in the spirit of a comment by Paul Kirkpatrick[4] in an early, seminal paper on the physics of baseball, "Our aim is not to reform [baseball], but to understand it," we look at a few aspects of the game.

The flight of the ball

The aerodynamic forces on the baseball are of the same magnitude as gravity, and to understand the flight of the ball we must know something of the aerodynamics of spheres passing through fluids. Following Fermi's advice on approximations, we estimate the drag force on the ball, with cross section A, as about the force required to give the cylinder of stationary air of density ρ ahead of the moving ball the velocity v of the ball. From this model, the drag force F_d will be $(C_d/2)A\rho v^2$, where $C_d/2$ is a dimensionless proportionality constant (the ½ is a convention) that we might guess would be somewhat less than 1 because some of the air will slide around the spherical ball before reaching the ball velocity.

This is a good model. For table-tennis balls and for baseballs traveling less than 60 miles per hour (27 meters per second), $C_d \approx 0.5$ over a span of velocities that covers more than two orders of magnitude. For baseballs traveling faster than 120 mph (54 m/s), for dimpled golf balls and for tennis balls hit hard by professional players, $C_d \approx 0.3$ and again does not vary strongly with velocity.

But baseball is played largely at velocities greater than 60 mph—about the initial velocity of a ball tossed softly from third base to first—and less than 120 mph—the initial velocity of the longest home run of the season. Hence baseball velocities fall between the two aerodynamic regimes, and the complexities of that interregnum

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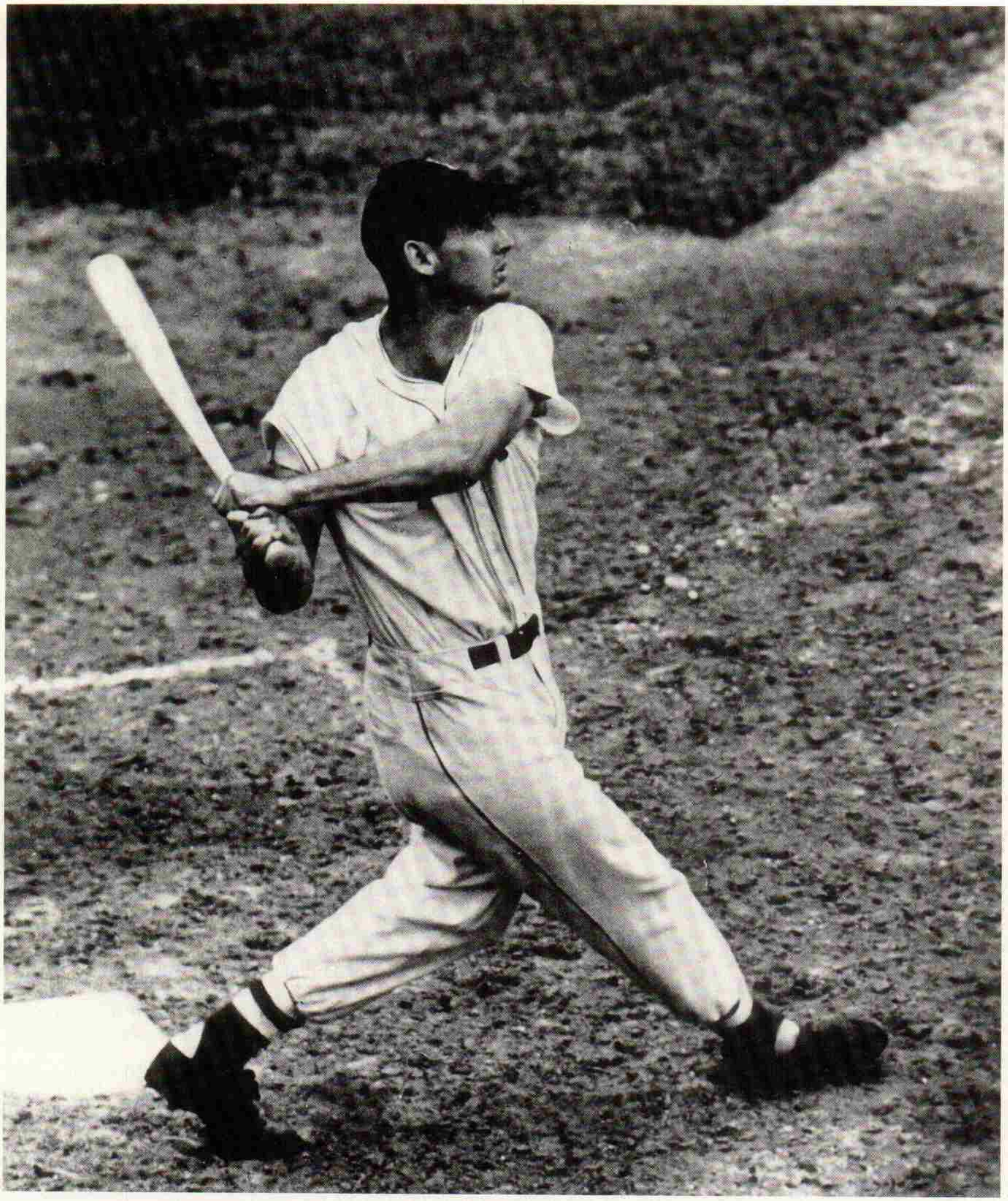
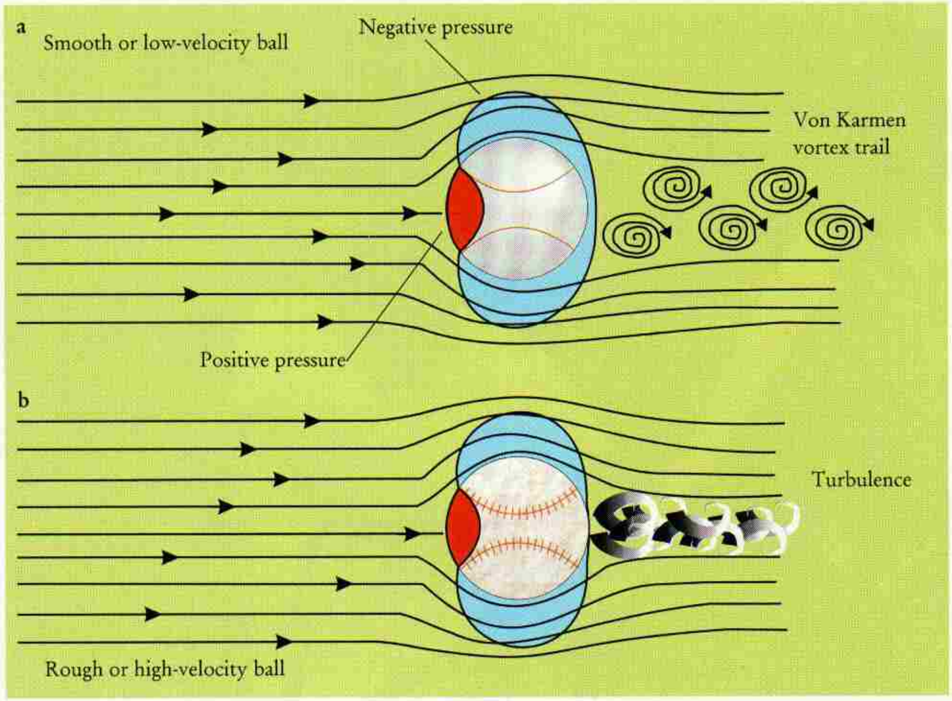


A NEARLY PERFECT SWING. Ted Williams of the Boston Red Sox was reputed to have one of the most efficient swings in baseball. After over 100 years of trial and error, the techniques of experienced professionals like Williams must be considered very nearly optimum, and so help to define the physics of the game. (Photo from the National Baseball Library and Archive, Cooperstown, New York.) **FIGURE 1**

enrich the game.[5] The air passes around the slow balls rather smoothly, held away from the ball by a layer of still air called the Prandtl boundary; behind the ball the airstream curls off in classical vortices. A very small insect, say a plant aphid, sitting on a ball sailing through the air at 50 mph (22 m/s) would scarcely note a breeze. The same bug would have to dig in against the wind and hold on for dear life on a ball traveling at 120 mph, when the boundary layer is largely blown away and the air behind the ball is turbulent.

The drag crisis—the transition between the smooth and turbulent regimes—occurs at lower velocities for rougher spheres. For a very rough ball the size of a baseball, the transition may take place at a velocity as low as 25 mph (11 m/s); for a very smooth ball, the transition might be delayed until 175 mph (78 m/s).



AERODYNAMIC FORCES on a ball moving through air depend on both the ball's velocity and its surface roughness. **a:** Low-velocity balls, regardless of surface roughness, pass smoothly through the air, generating an area of positive pressure in front of them and an area of negative pressure along their surfaces as the air speeds up to go around the ball. This area of negative pressure extends to the rear of the ball, generating a significant drag force. In the ball's wake, the air swirls in classical von Kármán vortices. **b:** At some velocity, whose value decreases as the ball's surface roughness increases, the flow of air around the ball becomes turbulent. Such flow still generates a positive pressure at the front of the ball. However, the area of negative pressure at the sides and rear of the ball is reduced. Thus, counterintuitively, the drag force is less for a rough ball than for a smooth ball. **FIGURE 2**

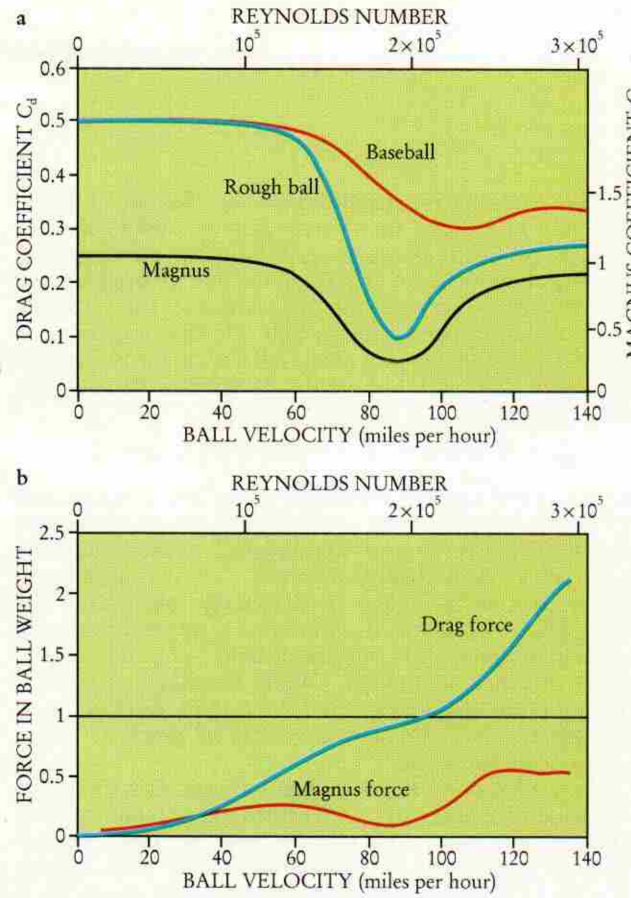
Figure 2 suggests the character of the air flow and pressure patterns about a rough and smooth sphere when the velocity is in the transition region. The pressure is positive on the front of both balls, negative, as per Bernoulli, as the air speeds up to go around the ball, and negative at the rear of the smooth ball. The negative pressure behind the smooth ball results in a significant drag. Thus, counterintuitively, the drag force on the rough ball is less, because the negative pressure there is minimal and encompasses a smaller area. For rough or smooth balls the drag crisis looks about the same.

But where does a baseball, with its rough stitches and smooth cover, fit in? It seems that for different orientations of the ball, different sectors of the stitching catch the air and induce drag-crisis transitions at different velocities. Figure 3a, shows the variation of the drag coefficient with velocity for a specific uniformly rough ball, where the transition takes place at about 70 mph (31 m/s), and an estimate (made in the light of wind tunnel measurements) of an average effective drag coefficient of a rotating baseball with its changing orientations. Figure 3b shows the drag forces for a baseball derived from the drag coefficients of figure 3a. Note that at about 95 mph (42 m/s), the drag force is equal to the force of gravity.

With these recipes for the drag force, together with a small correction for typical backspin, one can calculate the trajectories of balls hit or thrown with a given initial velocity and projection angle. The quadratic dependence of distance on velocity for projectiles in a vacuum is modified by air resistance to an almost linear relation for baseballs with velocities over 70 mph (31 m/s); for such velocities the maximum distance varies approximately as $200 + 5(v - 70)$, where the distance is in feet and the velocity in miles per hour. Thus a ball traveling with an initial velocity of 110 mph (49 m/s) will travel about 400 feet (122 m) instead of the 809 feet one might expect in a vacuum. Also, the angle of projection for maximum distance, rather than being the 45° optimum for a vacuum, is reduced to about 35°.

Home runs account for about 30% of the runs scored in baseball, and it seems that the probability of balls going over the fence varies as about the tenth power of the distance balls are hit. Hence the variation of that distance with the air temperature and altitude is important. Since the drag varies as the density of the air, it might seem simple to calculate the effect of altitude and temperature. If the drag coefficient were independent of altitude and temperature (as it is for a golf ball), the baseball would travel about 2% farther for every 1000 feet (300 m) of altitude—thus increasing the home run probability by about 20%—and about 0.5% farther for every 10 °F (5.5 °C) increase in temperature. But broadly speaking, the drag coefficient scales with Reynolds number $R = dv\rho/\eta$, where d is the ball diameter and η the viscosity. Hence at higher altitudes and temperatures, the drag coefficient curves of figure 3a move to the right, reducing the variation of the drag with altitude and temperature by perhaps a factor of two. But since that reduction of the variation depends on the precise shape of the drag coefficient curve in the transition region, which we don't

DRAG ON A BASEBALL. The drag force on a uniformly rough ball of cross-sectional area A moving with velocity v through air of density ρ is approximately $(C_d/2)A\rho v^2$. **a:** Baseball velocities are typically between 60 and 120 miles per hour, where the transition to turbulent flow—or "drag crisis"—causes C_d to vary rapidly with velocity. A rotating baseball is neither uniformly smooth nor rough, since it presents both its smooth cover and raised stitching to the air. This smooths the transition somewhat. However, a ball of radius r spinning with angular velocity ω interacts with air of density ρ through which it passes, generating a Magnus force equal to $C_m \rho A \omega r v/2$ perpendicular to the direction of motion and the axis of the spin. This is the force that makes a curveball curve. Below 60 mph the Magnus coefficient C_m is effectively constant with a value of about 1. The values are not well known at higher velocities; hence the results shown on the graph should be considered as sensible estimates. **b:** The drag force (calculated using the coefficients from **a**) increases monotonically and becomes equal to the force of gravity at about 95 mph. The Magnus force for $\omega = 1800$ rpm is always less than the force due to gravity but is still significant. **FIGURE 3**



know very well, our estimates of the effects of altitude and temperature are uncertain by about a factor of two. Even simple matters are not always easy.

The curveball

As tennis players noted long ago, spinning balls curve. Why? The 23-year-old Isaac Newton answered that the court-tennis balls curve because the side of the ball that moves fastest through the air meets more resistance than the side that moves more slowly. For a simple—and simplistic—description of a complicated process, that will still do as well as any other. When the right-handed sandlot pitcher throws a wide, breaking curveball to the plate such that it rotates at a rate of about 1800 rpm about a vertical axis and travels at a mean velocity of 70 mph, the side toward third base is traveling forward at a top-to-bottom average speed of about 80 mph (36 m/s), while the side toward first base is only moving at 60 mph (27 m/s). In Newton's description, the larger drag on the third-base side translates to a larger force—or pressure—and the ball swerves toward first base. (The big-league pitcher throws a tactically more effective curve, with more overspin and less sidespin, which then drops more but curves less.)

Physicists usually find it more congenial to discuss the spinning ball in the wind tunnel system where the ball is stationary and the air is moving. Then the third-base-side surface of the ball impedes and slows the flow of air around the ball, and the magnitude of the negative pressure at the side of the ball (see figure 2) is reduced according to Bernoulli's principle. The area over which the negative pressure extends is also reduced, as the airstream leaves the ball a little earlier. On the first-base side the air is speeded up and the negative pressure is increased (relative to that for the slower surface–air interface speed of the nonspinning ball). The air current also extends further around the ball, and the negative-pressure area is larger. And because the airstream is carried further around the ball on the first-base side than the third-base side, the airstream behind the ball is directed a little toward third base. Pressure difference or conservation of momentum, it's the same physics with different words: The ball curves toward first base, crossing the plate—60 feet 6 inches (18 m) from the pitcher's mound—as much as a foot and a half (0.5 m) from where the noncurving ball would have crossed, and the sandlot batter swings where the ball isn't and misses.

The curveball effect is called the Magnus effect and the force causing the curve the Magnus force, after Gustav Magnus, who measured the phenomena about 1850.

If the pitcher threw a smoother ball a little faster and with a little more spin, the faster, third-base side of the ball would initiate the drag crisis and blow away the boundary layer, causing the air on that side to hug the ball more closely for a longer distance around the perimeter of the ball (as shown in figure 2), thus leaving a low-pressure area over a large region and creating a turbulent wake. On the slower, first-base side the air would still flow smoothly around the boundary layer and separate from the ball surface relatively early. As a consequence, the total pressure force on the ball would be toward third base, the air around the ball would be deflected toward first base, and the ball would curve toward third base! Lyman Briggs saw such an "inverse Magnus effect" for smooth balls in the course of wind tunnel experiments in the 1950s.[6]

For rotating baseballs showing different aspects to the air, the drag crisis seems to be smoothed out—as shown in figure 3a—so nothing so dramatic as a reversed curve shows up. Smooth golf balls traveling from a solid hit off the tee with velocities in the transition region and with a backspin of about 3600 rpm duck into the ground as a consequence of such an inverse Magnus effect and won't go a hundred yards (90 m). But when properly roughened by dimples, the ball travels in a wholly turbulent regime, and with a normal Magnus effect, where the backspin generates lift, the low-trajectory drive travels for long distances.

F [sub m]

These qualitative insights into the Magnus effect do not lead unerringly to a quantitative understanding. However, we can make some useful guesses starting from Newton's description of the Magnus force as proportional to the difference between the drag forces on the slow side and fast side of the spinning ball. If the drag force is proportional to the square of the velocity, as it is for balls at velocities below and above the drag crisis, the Magnus force can be described by the relation

$$F_{\rm m} \propto [F_{\rm d}(v + \omega r) - F_{\rm d}(v - \omega r)] \rightarrow C_{\rm m} \rho A \omega r v / 2$$

where ω is the angular velocity, r is the ball's radius, and the Magnus coefficient $C_{\rm m}$ is determined to be about 1 at low velocities according to experimental measurements by Robert Watts and Ricardo Farrar.[7] The variation with velocity of that coefficient for baseballs taken from the above relation is shown in figure 3a, and the variation of the Magnus force with velocity of balls spinning at 1800 rpm is shown in figure 3b. While the Magnus force is less than the drag force and less than gravity, it is sufficient to move the slow curveball as much as 2 feet on its trip to the plate.

This expression for the Magnus force and the curve of figure 3b for baseballs are verified experimentally only below 50 mph. But the relation does account, qualitatively, for the negative Magnus effect for balls with a uniform surface and also fits what we know about baseball. For example, the forces from figure 3b explain the break in the 65-mph curveball quite well and show why the tailing 90-mph fastball breaks only about 4 or 5 inches (10 cm). If there were no dip in the Magnus coefficient at 90 mph, we could expect the fastballs to tail off—and hop—much more.

How much does the spin of a ball affect the drag? Measurements disagree. But from the laboratory defined by the games, I conclude that the drag on a ball is not much increased by its spin; otherwise I can't account for John Daly's 300-yard (275 m) golf drives. And how much does the reaction from the forces that cause the spinning ball to curve slow down that spin? If I am to understand the trajectories of pop flies, the reason home-run hitters swing up at balls and why, after my right arm was disabled in World War II, I could catch fly balls barehanded with one hand, but not foul balls, I have to conclude that the spin falls off quickly in time. The physicist's model of the game must fit the game.

The knuckleball

In a wind tunnel, wires placed in an appropriate position on the surface of a sphere can trip the drag-crisis transition at relatively low velocities. So when a pitcher throws a knuckleball off the fingertips at a velocity near 60 mph, with so little spin that the ball will rotate no more than one-half revolution on the way to the plate, the raised stitching can catch the air on one side of the ball and trip the transition to turbulence while the air continues around the boundary layer on the other side. The ball then veers toward the stitch that catches the air. Sometimes, if rarely, the ball will rotate such that the stitches on one side will trip the transition early in the flight to the plate and then stitches on the other side will take over. Watts and Eric Sawyer[8] have shown that such a ball can execute a double curve!

The asymmetric forces that are generated are considerable, so the ball breaks so sharply that it is almost impossible to hit and very difficult even to catch. But even the most skillful pitcher has great difficulty in throwing the ball with the precision required to generate a reproducible break, so the pitch is too often a surprise to everyone—batter, catcher and pitcher.

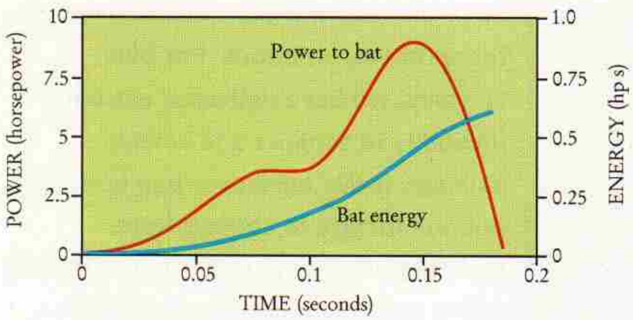
Batting and throwing

The study of the complex interaction of muscle, tendon and bone that underlies the swing of the bat is a game best played by physiologists, not physicists. But a physicist can limn the character of the swing through simple analyses of the variation with time of the energy transferred to the bat. Although nothing as elegant and complete as Ted Jorgenson's study of the golf swing[9] is available for baseball, simple arguments still lead to some interesting conclusions. We proceed by modeling the swing and analyzing the model.

The modeling is made possible by the character of the swing; the bat is *swung* by the good batter—a little like a rock on the end of a string. The torque applied by the hands and wrists is negligible. I could thus establish—by trial and error—a time-dependent position of the hands that pull the bat about the familiar arc with a peak speed of the bat's "sweet spot"—the spot that transfers maximum energy from the bat to the ball—of about 70 mph, the velocity that would drive a ball for a long 380-foot home run. Checked with photographs and videos of players swinging a bat, the model is sufficiently good to allow one to draw reliable, broad conclusions.

It takes about 0.2 seconds from when the batter begins the swing until the bat crosses the plate. Hence the batter must begin swinging at the fastball when the ball is about halfway from pitcher to plate—though the batter can still hold back while the ball travels another 10 feet. As shown in figure 4, the peak rate of energy transfer to the bat, which occurs about 30 milliseconds before the ball is hit, reaches about 9 horsepower. Taking the maximum power generated by muscles as no more than 1 horsepower per 10 pounds (about a kilowatt per 6 kg), we see that the contribution of the hands and wrists cannot be important; the energy must come largely from the large muscles of the thighs and thorax. Even then, it is difficult to understand the energy transfer without postulating a storage mechanism. It seems that early in the swing the batter stores energy in the translation and rotation of the body, and that energy is transferred to the bat, by means of the strong arms of the hitter, in the 50 milliseconds before impact. From videos, one can see that the bodies of some especially efficient batters are almost motionless when the bat hits the ball; the centrifugal force of the bat exerted through the arms of the batter has stopped the batter's motion—all of the energy has gone into the bat.

The energy transfer in pitching is even harder to understand to one's satisfaction, but again a simple model provides useful insights. Assuming the ball is accelerated at a constant rate through a distance of about 8 feet (2.5 m), a force of about 10 lbs (45 N), giving an acceleration of about 40 times gravity, is required to throw the ball with an initial velocity of 97 mph (43 m/s), so that it crosses the plate at 90 mph (40 m/s), the speed of a typical major-league fastball. The average energy transfer over



BATTING POWER. A batter's swing typically lasts 0.2 seconds, during which time the rate of energy transferred to the bat increases from 0 to about 9 horsepower during the first 0.15 seconds and then decreases to 0 as the bat crosses the plate. Because muscle can generate only about 1 horsepower per 10 pounds, the majority of the swing's power must come from the large muscles of the legs and thorax rather than from the hands and wrists. Even assuming a major contribution by these large muscles, the power of the swing can be explained only if the batter stores translational and rotational kinetic energy early in the swing and transfers that energy to the bat late in the swing. **FIGURE 4**

the 0.11 seconds of acceleration is about 1.5 horsepower (1.1 kW), meaning that the total energy of the hard-thrown ball is about one-third that of the hard-swung bat and is generated in about 60% of the batting time. More realistically, the peak power must be appreciably larger than 1.5 hp and cannot be generated by the arm alone. As in batting, the energy must come largely from the muscles of the thighs and thorax.

The bat and the ball

The character of baseball has evolved as a result of delicate balances—between hitter and pitcher, between bat and ball. Drop a baseball onto concrete from 10 feet and the ball will bounce up only about 3 feet; the coefficient of restitution is then about $\sqrt{3/10} = 0.55$. At higher velocities the ball appears to be even less elastic. A home run that sends the 90-mph fastball back with a velocity of 110 mph (50 m/s) generates that reversal in a very short time. If the 2.9-inch (7.35 cm)-diameter ball were crushed to one-half its diameter and acted as a linear spring, the collision would take about 2 milliseconds. But rough stress–strain measurements on the ball give the commonsense result that the spring is nonlinear—the force increases to about 9000 lbs (40 000 N) as the ball is compressed—and the total collision time is only about 1 millisecond, with most of the momentum transfer taking place in about 0.6 milliseconds.

Given the coefficient of restitution, various people have calculated the kinematics of the collision of the ball with the swinging bat assuming the bat is a rigid body with a given mass and moment of inertia, sometimes with a comment about neglecting the effect of the hands on the bat. But on the time scale of the collision, the bat is not at all rigid. If you tap a 34-inch (0.85 m)-long wooden bat with a light hammer, the bat rings with a note corresponding to a frequency of about 180 Hz. With a little more tapping, you can define the wavelength of the sound by finding the nodes on the bat. These are typically about 20 inches (0.5 m) apart. Crudely speaking, the velocity of transverse waves in the bat will then be on the order of 600 ft/sec (180 m/s) and it will take the collision impulse signal about 8 milliseconds to go the 5 feet (1.5 m) from the point of impact to the hands and back. But the ball will have long left the bat in that time and will never know whether or not hands were holding the bat. On the time scale of the collision, the bat is flexible. In the same vein, the ball will not know if weight was added near the handle; the moment of inertia of the bat is not relevant in any simple way to the collision kinematics.

In one of the more elegant calculations in sports physics, Lonnie Van Zandt has properly considered the flexibility of the wooden bat and the resultant complexities of collisions of the bat and ball.[10] Van Zandt's results may also have explained some bat manufacturing complexities that had puzzled me. I had doubted the standard wisdom that good bats could only be made from very special growths of American ash. Van Zandt's calculated spectrum of the fundamental and first 20 harmonics of his bat fit measured values to about 1%, but only after the handbook values for the elasticity of ash were changed to give a 25% correction. So I suspect that the bat was made with especially "good wood" and the manufacturers do know what they are doing.

Baseball rules require that the bats used in professional baseball be constructed of wood, whereas amateur players can use aluminum bats. Players agree that the aluminum bats drive the ball much farther; the balls come off the aluminum bat with more velocity. Why? When the ball hits the wooden bat, the bat compresses about 2% as much as the ball—and hence stores about 2% of the collision energy. The ball, with a coefficient of restitution at high velocities of about 0.45, returns about 20% of its 98% of the stored energy, while the bat, which is about as elastic as the ball, returns about the same proportion. By contrast, the hollow aluminum cylinder that forms the barrel of the aluminum bat is distorted about 10% as much as the ball by the collision and so stores about 10% of the collision energy. And it returns that energy efficiently—probably at a level of about 80%. Adding the ball and bat contributions, about 26% of the collision energy is returned, the ball leaves the aluminum bat with a higher velocity, and the 370-foot drive to the warning path by the wooden bat goes over the fence, for a 400-foot home run. Overall, the use of aluminum bats could be expected to double the number of home runs hit during a season. And that would change the balance of the game too much.

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